THE JOURNAL of the Acoustical Society of America

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Recording Lissajous Figures

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(Received October 9, 1944)

THE following article is not intended to contribute anything new on the theory of vibrating bodies or of Lissajous figures but it is a description of a refined method of using a double pendulum for the production of these figures and the making of them into a permanent record. The beauty and design of the many patterns produced should be of interest to the scientist and teacher, as well as to the artist, and all interested in unique design.

Lissajous curves or figures could be just as properly called Bowditch curves or figures since a Yankee, Nathaniel Bowditch of navigation fame, as early as 1815 produced similar curves by a pendulum suspended from two points. The *American Academy of Arts Science Memoirs*[1] carries an article by him and one by James Dean, Professor of Mathematics and Natural Philosophy, University of Vermont, in which were announced the invention and description of the double pendulum. It consisted of a Y-shaped affair having different oscillating lengths at right angles to each other and, therefore, different periods. It is not known just how Bowditch recorded the path of his pendulum, if at all, but

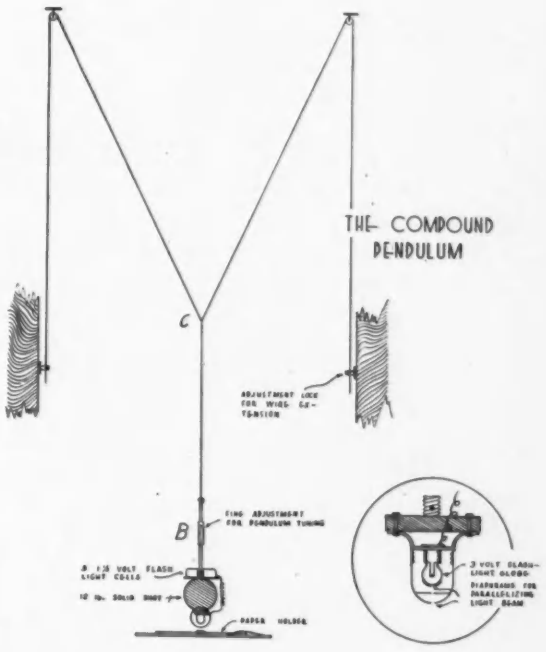

FIG. 1. Diagram of double pendulum and light housing.

* Now at Rutgers University, New Brunswick, New Jersey.

[1] *American Academy of Arts Science Memoirs* (1815), Vol. 3.

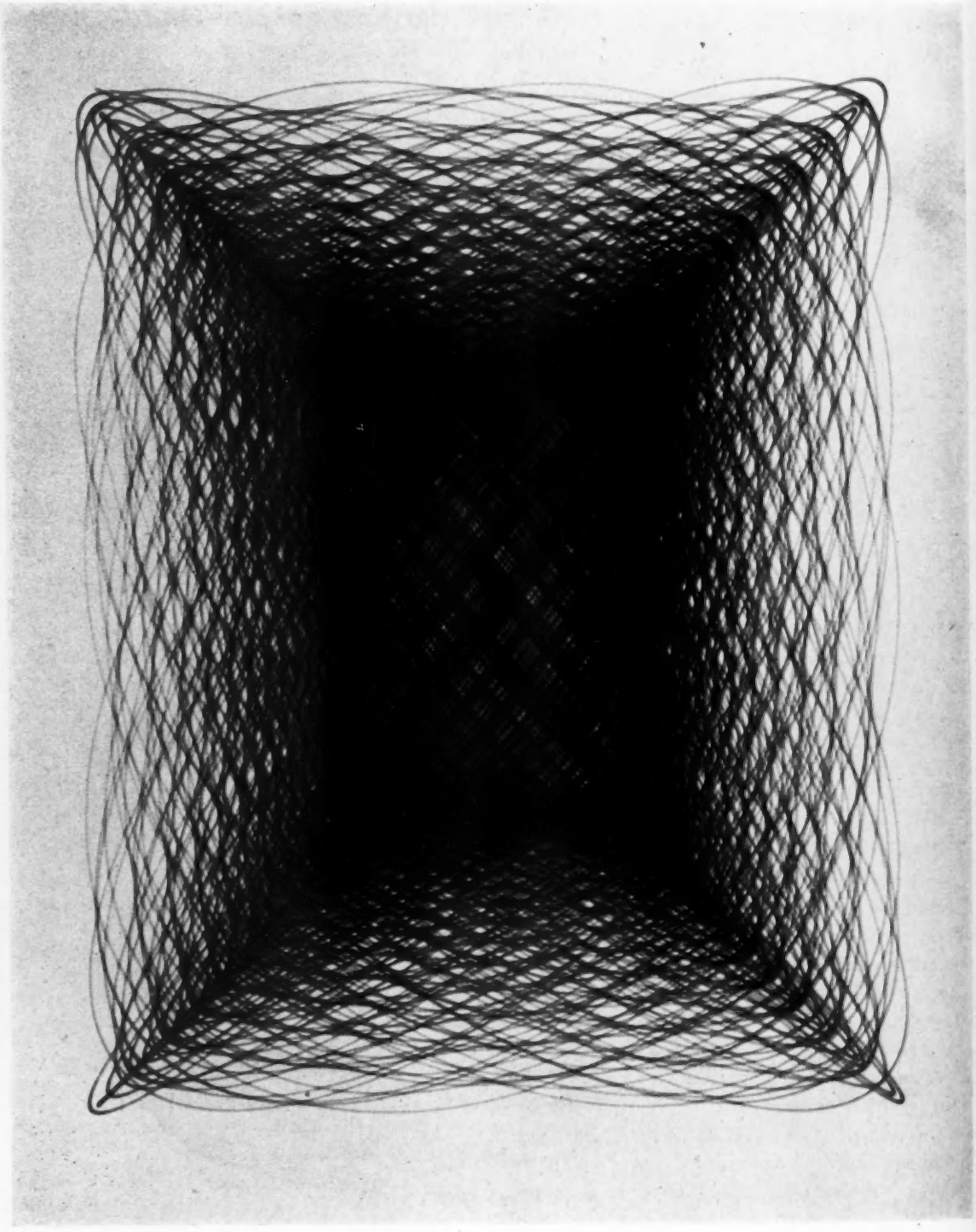

FIG. 2A. $r=7/6$. $T=3.66$ sec. $T_1=3.13$ sec. Total time of exposure = 25 min. Number of cycles = 68.2.

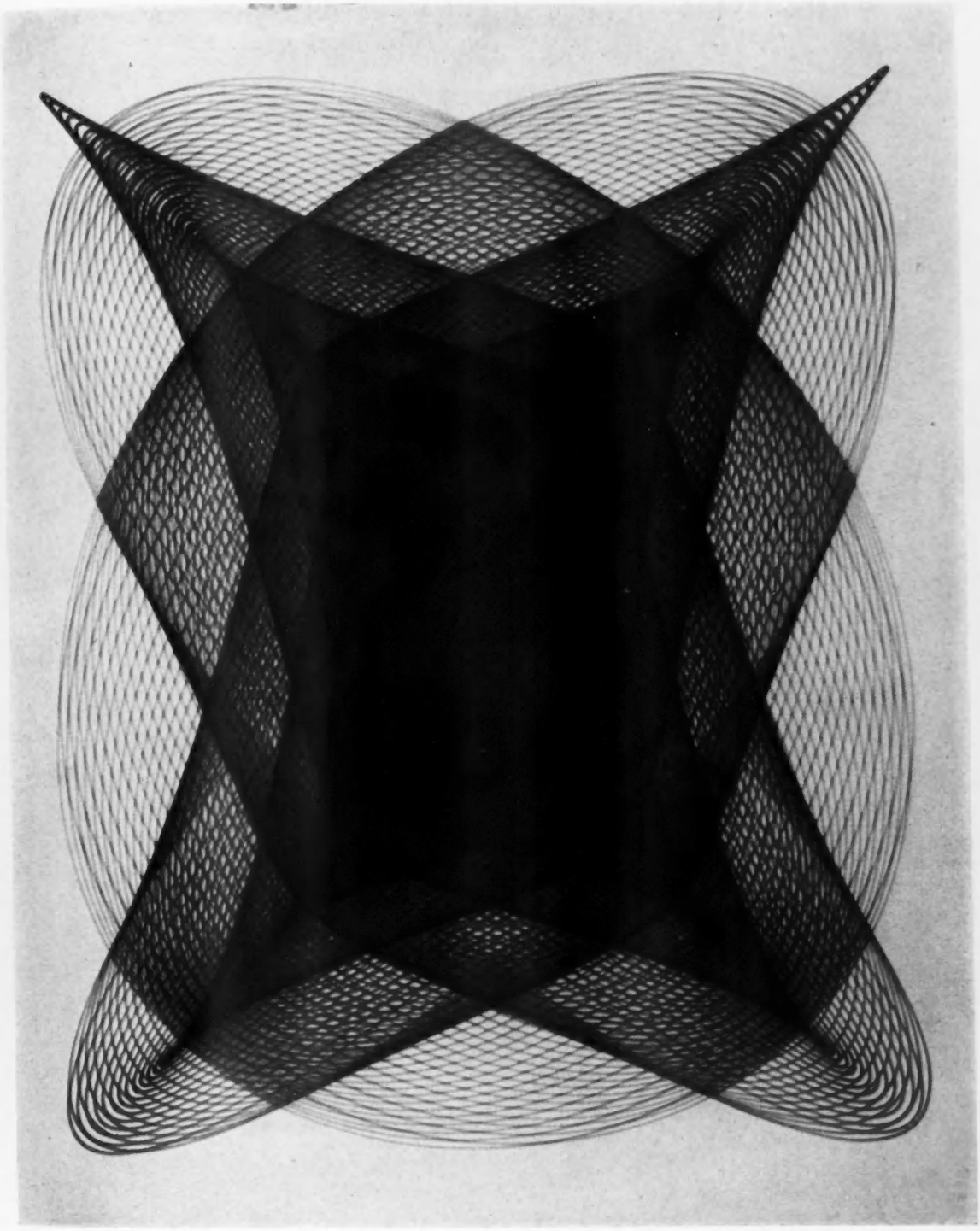

FIG. 2B. $r = 7/6$. $T = 3.648$ sec. $T_1 = 3.128$ sec. Total time of exposure = 32 min. Number of cycles = 87.0.

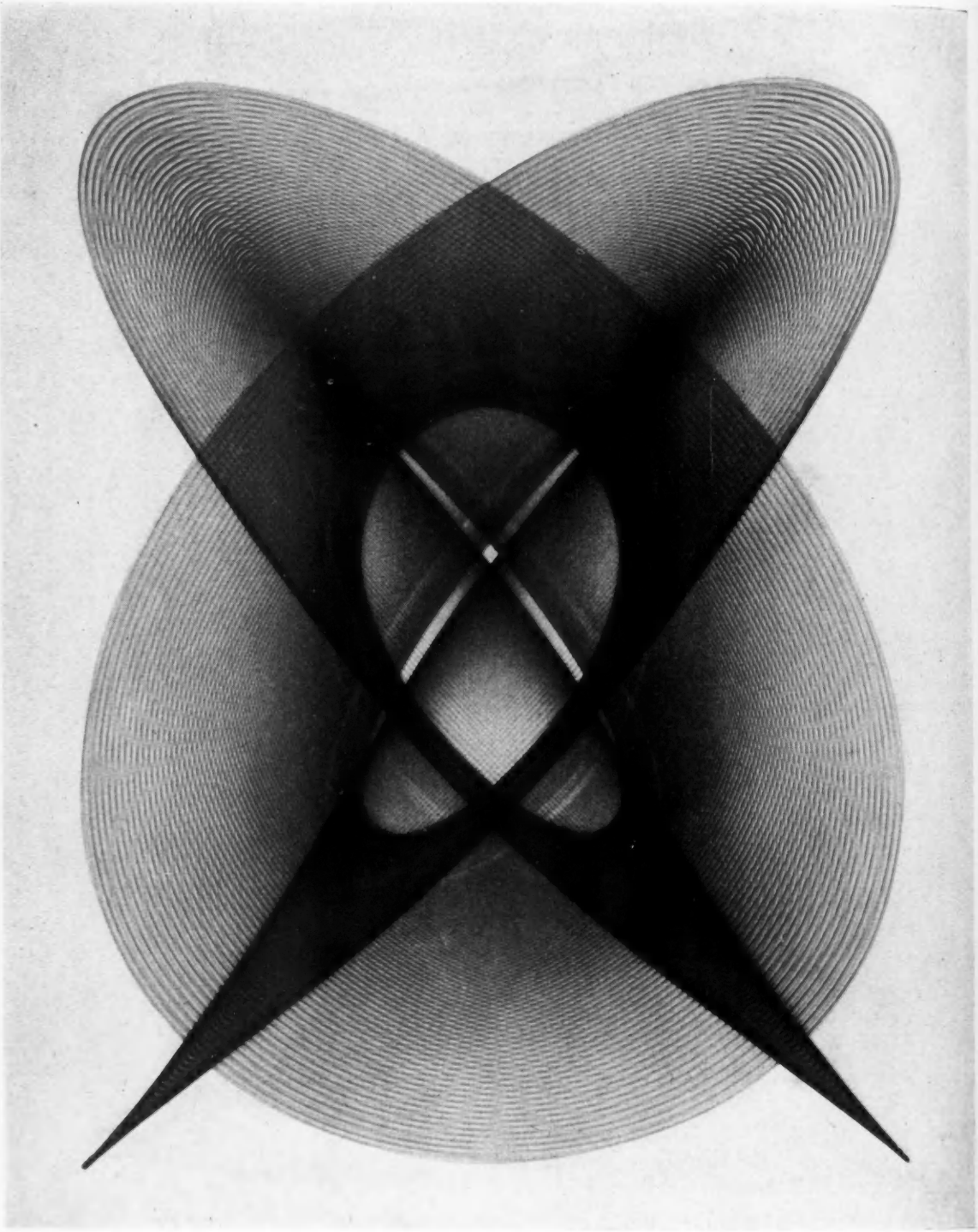

FIG. 2C. $r = 5/4$. $T = 3.650$ sec. $T_1 = 2.919$ sec. Total time of exposure = 24.5 min. Number of cycles = 100.

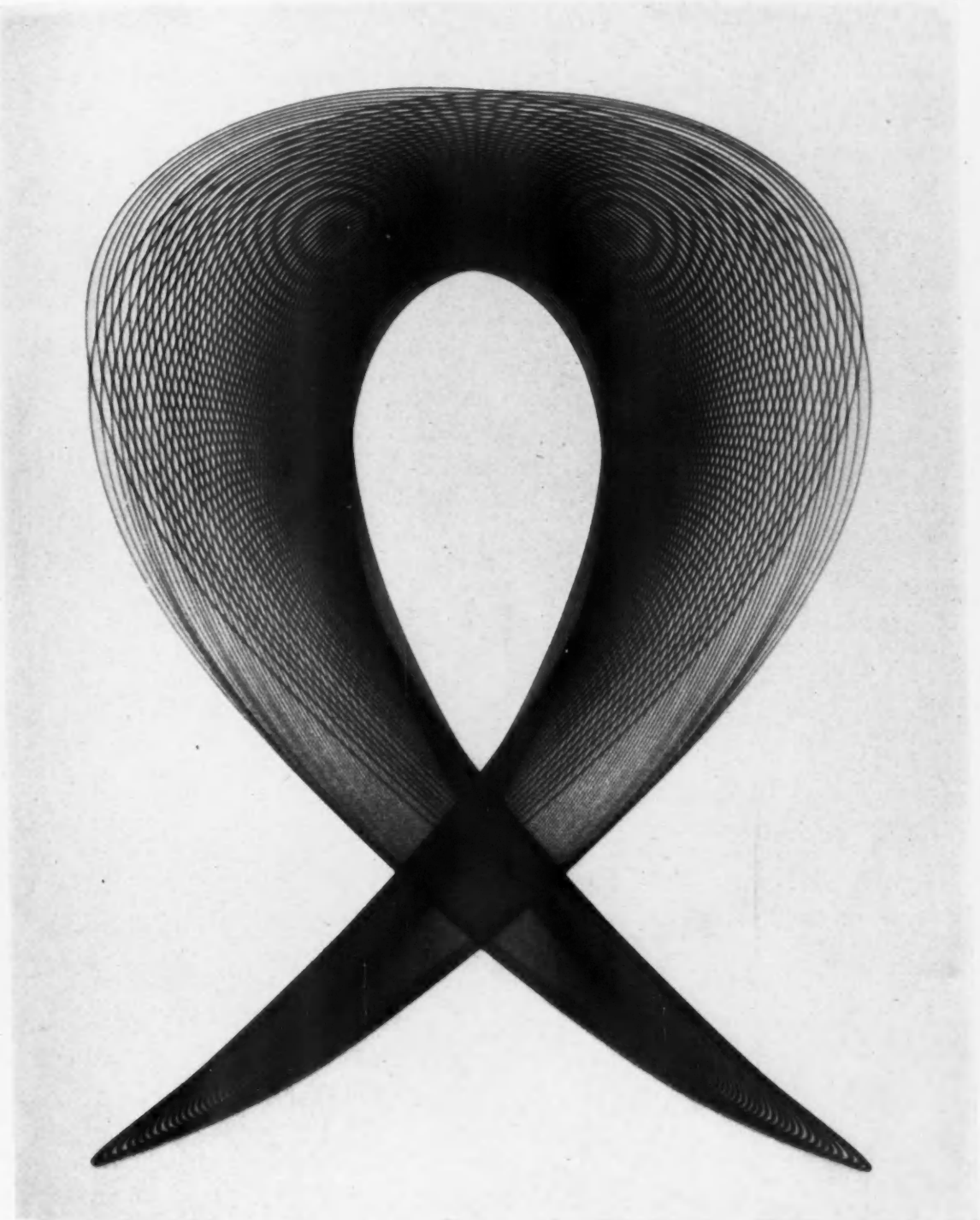

FIG. 2D. $r=3/2$. $T=3.650$ sec. $T_1=2.433$ sec. Total time of exposure = 7.3 min. Number of cycles = 60.

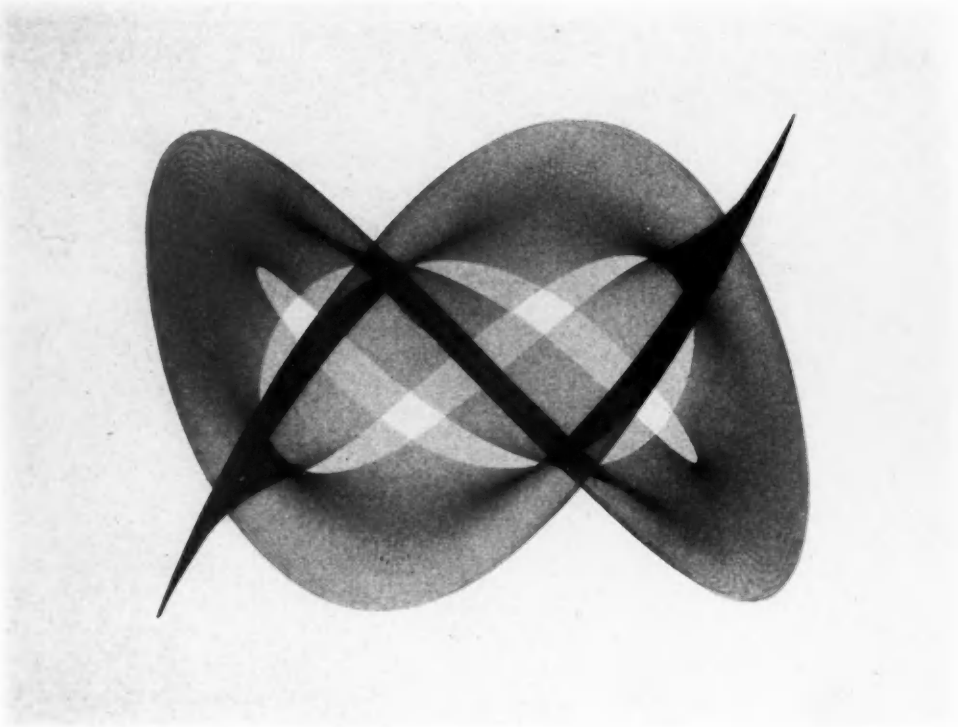

FIG. 3A. $r=5/3$. $T=3.700$ sec. $T_1=2.22$ sec. Total time of exposure = 30.0 min. Number of cycles = 162.

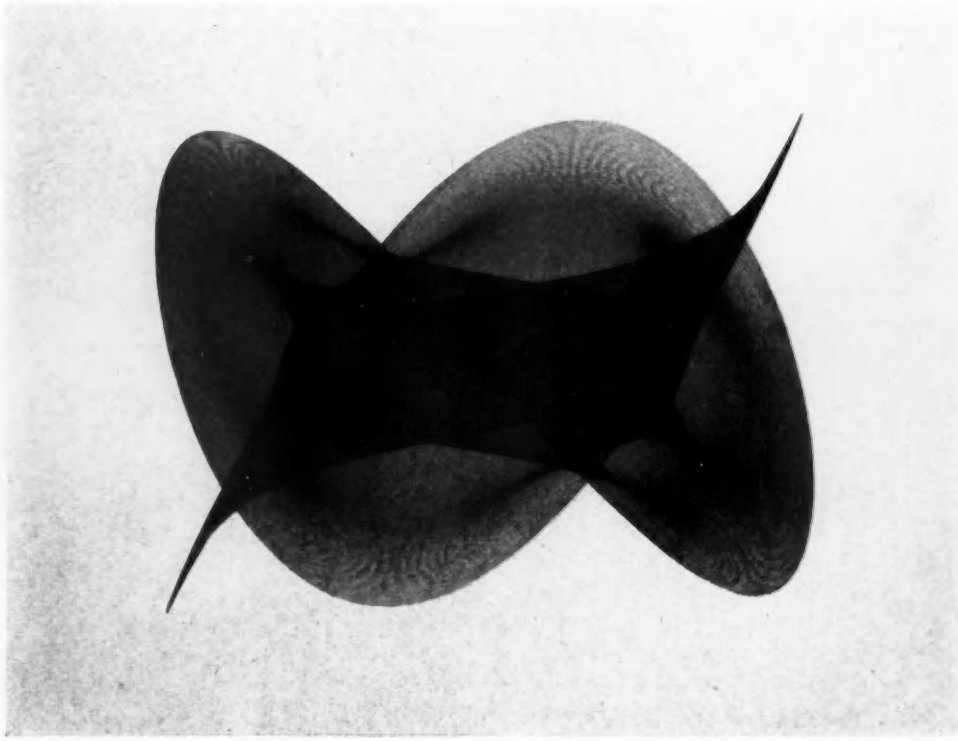

FIG. 3B. $r=5/3$. $T=3.700$ sec. $T_1=2.22$ sec. Total time of exposure = 60.0 min. Number of cycles = 324.

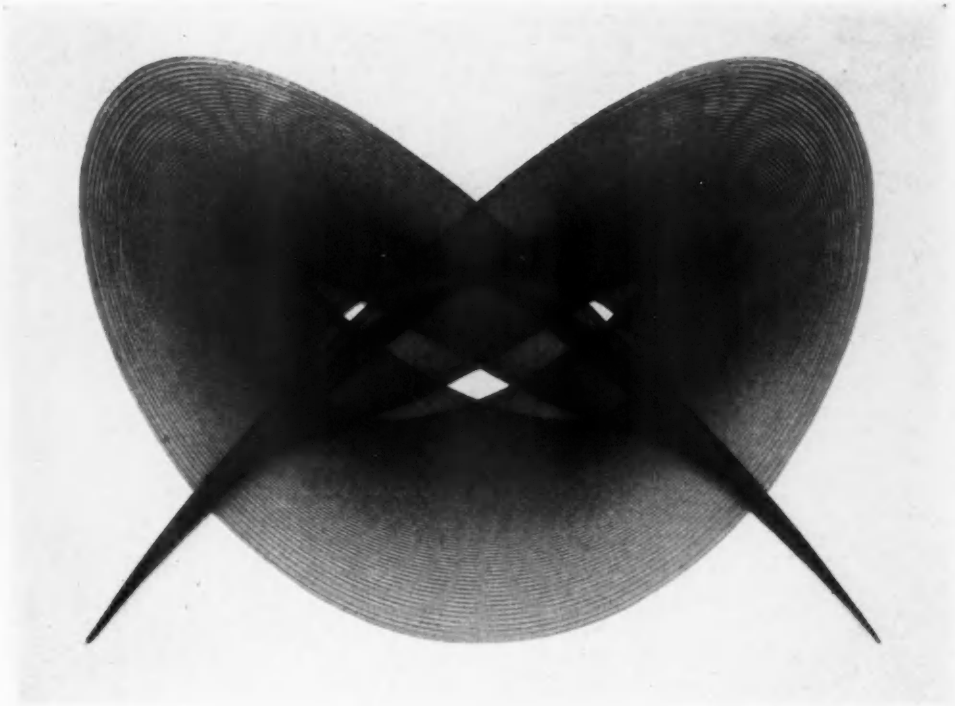

FIG. 4A. $r=4/3$. $T=3.650$ sec. $T_1=2.738$ sec. Time of exposure = 22.0 min. Number of cycles = 120.0.

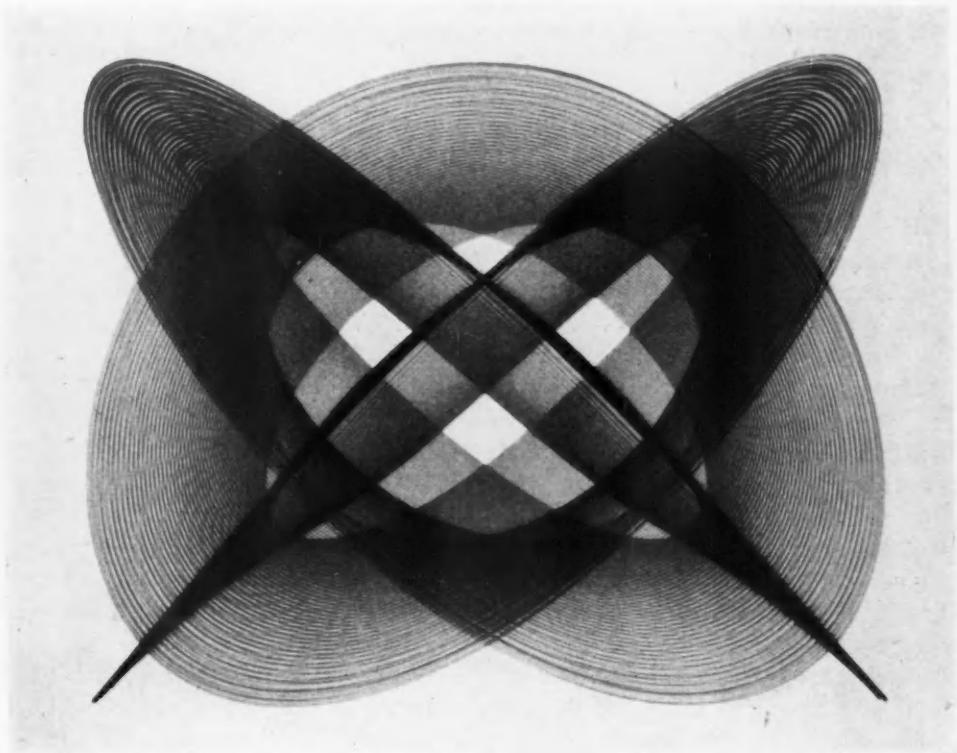

FIG. 4B. $r=6/5$. $T=3.650$ sec. $T_1=3.04$ sec. Time of exposure = 15.2 min. Number of cycles = 50.0.

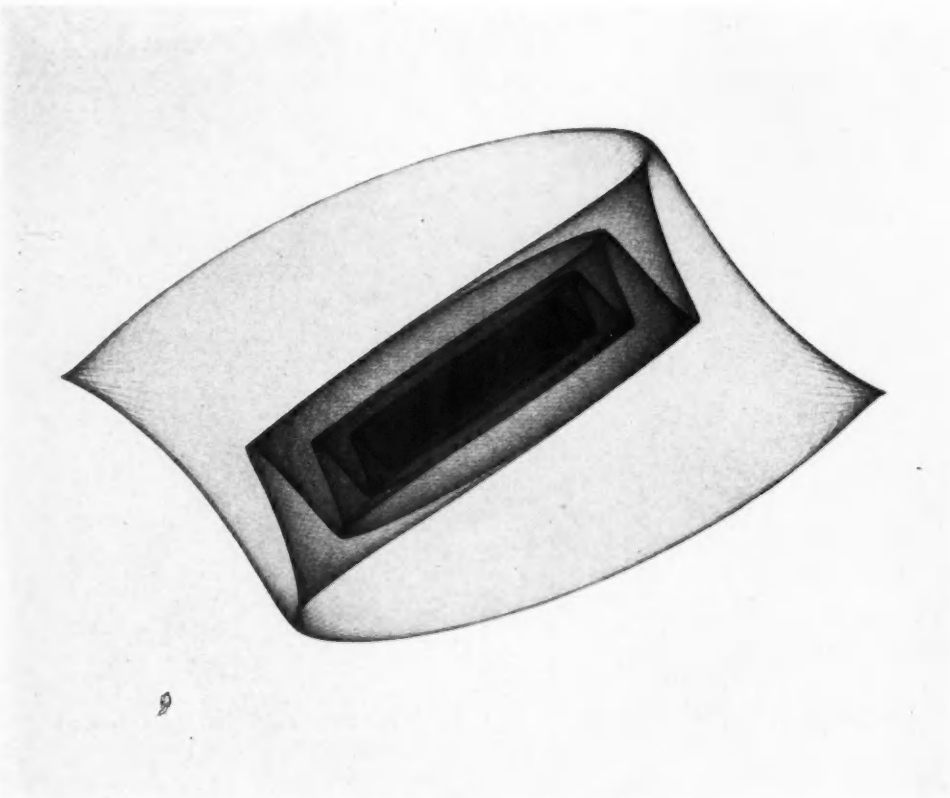

FIG. 5. This unusual pattern was obtained by holding the pendulum line to one side in a semirigid way and allowing the short length arm of the pendulum to oscillate for one hour as a Foucault pendulum.

about twenty-nine years later Hugh Blackburn of Glasgow invented a double pendulum having a conical bob, the apex of which had a small hole through which sand or salt could filter and leave a record of the track being described.

During the years 1856–58, Jules Antoine Lissajous, a teacher at the Lycée St. Louis in Paris, made extensive studies of these compounded harmonic motions at right angles to each other. He published two papers on this subject. This publication together with his more extensive lecturing and demonstrating of this phenomenon has resulted in his name being more effectively attached thereto. Lissajous used an optical method of producing these figures and this permitted them to be projected on a screen and observed by an audience of any size. His apparatus consisted of two tuning forks each carrying a small mirror and vibrating at right angles to each other. A spot of light is reflected successively from the two mirrors onto a screen where the combined motion of the two forks is clearly shown. If the vibrations are fast enough, the peristence of vision enables the observer to see a continuous curve.

Many methods have been designed for reproducing these figures—such as the Harmonograph of A. E. Donkin and the Kaleidophone of Sir Charles Wheatstone.

The late Professor Wallace Clement Sabine of Harvard drew some very beautiful curves by mechanical means. Eight of them are reproduced in Wm. F. Osgood's *Mechanics*, and according to Professor Osgood, this is the only reproduction that has been made of Professor Sabine's drawings. The details of his mechanism are not known by this author. Numerous other reproductions are found in *Harmonic Vibrations and Vibration Figures* by Goold, Benham, and others, and in *Harmonic Curves* by William F. Rigge.

The most modern way of producing these figures is by means of the cathode-ray oscil-

lograph. In this instrument the vibrations at right angles to each other are electrical and produce proportionate displacements of a beam of electrons which subsequently falls on a photographic plate or phosphorescent screen where the resultant Lissajous figure is recorded. The formation of such figures with frequencies up to at least 10 megacycles has been produced.

The equations of motion for these curves are given by

$$X = a \sin\left(r\frac{2\pi t}{T} + \alpha\right),$$

$$Y = b \sin\frac{2\pi t}{T},$$

where a and b are the respective amplitudes of vibration, T the fundamental period of vibration, α the phase difference in the two motions, and r a constant representing the ratios of the periods of the two vibrations.

$$r = T/T_1 = (L/l)^{\frac{1}{2}},$$

where

$$T = 2\pi(L/g)^{\frac{1}{2}} \quad \text{and} \quad T_1 = 2\pi(l/g)^{\frac{1}{2}}.$$

When $r=1$ and $\alpha=0$ the motion is a straight line which may be changed to a circular or elliptical path by introducing a phase angle α of the desired value.

As r increases in simple ratios, the complexity of the motion becomes much greater and an analytical treatment of the problem by eliminating t and obtaining X in terms of Y likewise becomes greater. When $r=2$, the motion describes a quadratic path; when $r=3$, the path will be a cubic. Beyond this degree the analytical analyses of these curves become very involved and they are usually studied by graphical methods. A complete discussion of the theory connected with these curves will be found in any good treatise on sound or mechanics.

When the frequencies are very near whole numbers, the differences have the effect of introducing a small or large phase angle between the vibrating components and, as a result, there is a migration of the cusps of the figure being described. This migration will be rapid or slow depending on the accuracy of the tuning or the integral relationship of the ratio. If, for example, the frequency ratio is 4/3, the short length pendulum will make four complete vibrations while the longer one makes three complete vibrations, and they return to the point of origin together to begin another cycle. If, however, the ratio is 4.01/3, then the shorter one will arrive at the point of origin just a little ahead of the longer one and they will be out of step. This lead will increase as time goes on until the short pendulum is a half-vibration ahead of the longer one and the cusps will appear on the opposite side of the sheet and the vibrations appear to be in step again. This continues through the period of the exposure. Figure 3A is a splendid example of sharp or accurate tuning. This exposure continued for 30.0 minutes and the two vibrations were in step even at the end. During this time 162 complete cycles were executed and the bob came back to the initial point at the end of each cycle. Figure 2B is an example of near tuning in which the cusps migrated from corner to corner during the 32.0 minutes of its exposure. The basket weave of Fig. 2A is a result of very poor tuning. The frequencies were so incommensurate that they were never together after the release of the bob at the beginning of the exposure.

The present method of recording Lissajous figures was begun several years ago in the Physics Laboratory of Brigham Young University. A number of students have worked on the project, each contributing refinement which added to the precision of timing and recording and to the beauty of the resulting pattern. Professor Charles A. Slichter of Wisconsin used a similar method (unknown to us during the development of our method) of making stereoptican photographs of single cycles of harmonic curves of three frequencies.[2]

The apparatus in its final form consisted of an ordinary Blackburn pendulum, the principal length L of which was 325 centimeters. The bob consisted of a heavy brass ball—weight twelve pounds—on which was constructed a rack to carry two dry cells and under which was constructed a housing for the flashlight globe, and

[2] His work is described in *Transactions, Wisconsin Academy of Science, Arts, and Letters* (1896–1897), Vol. 11.

a series of three pinholes in line to insure parallel light. A piece of 11×14 inch double weight Kodabromide F5 paper was held in place by a printing board of convenient size. The bob was tied back to one corner of the paper and released simultaneously with the turning on of the light and starting the chronometer. All this was done in the red light of an Eastman safe light for bromide paper. The exposure was allowed to continue for a few seconds for one cycle or for an hour and a half for 450 cycles or more.

The pencil of light is fine enough and the damping of the bob sufficiently large to record every path as a fine distinct line, so that areas of light and dark are visible to produce the fine structure of the pattern. Even the black areas where the damping is small near the end of the exposure show a unique and fine structure detail and symmetry in the originals.

One sees, therefore, that an infinite variety of designs or patterns could be produced by varying any one of a number of factors which include frequency ratio, phase, amplitude, damping, and time of exposure. Much time was always consumed before each exposure to insure perfect timing for each ratio of periods recorded. It was found that ratios had to be synchronized by the method of timing because direct measurements of respective lengths proved quite inaccurate.

Rough adjustments to change the period or frequency ratios were made by means of the clamp *C*, Fig. 1, until the ratio of lengths was the desired value. Finer adjustments were later made by timing the periods in each phase of motion and making adjustments at the turn buckel *B*.

Since we began recording Lissajous figures by this method, nearly 200 exposures have been made and 150 different and attractive patterns have been produced—no two of which were alike. Sixty of these were exhibited in the foyer of the Hotel Pennsylvania during the May, 1944 meeting of the Acoustical Society of America; nine of them are reproduced with this article.

On the Theory of the Directional Patterns of Continuous Source Distributions on a Plane Surface

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(Received September 1, 1944)

This paper contains a study of the directional patterns which may be obtained by driving a plane surface with specified normal velocities. Part I is devoted to the general theory of such distributions. The directional pattern at any distance from an arbitrary continuous distribution of sources is first obtained. The approximate relation which holds at large distances is then derived, and the simpler formulae which hold when the surface distribution has several different kinds of symmetry are developed. In particular, the case is studied in which the sources are distributed along a line. An interesting relation between linear distributions and those with circular symmetry is pointed out. Part II contains the derivations of a number of directional patterns corresponding to specific source distributions. It is pointed out that although it is easy to specify a source distribution which will yield any directional pattern satisfying certain methematical conditions of convergence, such distributions are usually of infinite extent, and therefore do not correspond to distributions which are of practical interest. A method of attack is suggested for the problem of determining the distribution of specified extent which yields the directional pattern of optimum characteristics for any particular application. Part III contains 22 tables, 21 of which are tables of the directional patterns of the source distributions considered in Part II. In addition to the column giving the argument, each of the 21 tables contains three columns, the first giving the relative sound pressure, the second the square of the relative sound pressure, and the third, ten times the common logarithm of the inverse square of the relative sound pressure. Table II, which is more detailed than the others insofar as the interval is smaller, is a table of the directional pattern of a circular piston.

INTRODUCTION

A NUMBER of papers[1–3] have been written on the subject of the theoretical directional patterns of sound radiators. The subject is very closely related to the problems of the directionality of antenna arrays[4–6] and of optical systems.

In his first paper, Stenzel has a detailed examination of the properties of the linear array of point radiators of equal amplitude, and discusses the possibility of shifting the direction of the principal lobe by letting the phase of each radiator differ by a constant amount from its neighbor on one side. In his second paper, which is concerned chiefly with surface distributions, Stenzel derives a general formula for the curvature of the directional pattern at the peak of the principal lobe in terms of the principal "moments of inertia" of the surface distribution. Stenzel's most important contribution is undoubtedly his discussion in the second paper of circular arrays of point radiators of equal amplitude. Both of Stenzel's papers contain many polar diagrams of the directional patterns which are discussed.

Menges[3] has attacked by brute force methods the problem of reducing the magnitude of the minor lobes of a continuous linear array. He exhibits a distribution with a pattern whose minor lobes are all more than 40 db down from the principal lobe. The efficiency of this distribution as defined by Eq. (64) is slightly less than one-half.

The nature of the present paper is described in the abstract.

PART I. GENERAL THEORY

Consider an infinite plane surface on which is erected a rectangular Cartesian coordinate system. The x and y axes lie in the plane, and the z axis, therefore, is normal to the plane. The x, y, and z axes are to form a right-handed coordinate system.

Consider further that each point of the surface executes harmonic vibrations about its equilibrium position, and let the normal velocity at the point x,y be indicated by $\mathfrak{A}(x,y)e^{i\omega t}$. The amplitude $\mathfrak{A}$ is a complex number; the absolute value of

[1] H. Stenzel, E. N. T. **4**, 239 (1927).
[2] H. Stenzel, E. N. T. **6**, 165 (1929).
[3] K. Menges, Akustische Zeits. **6**, 90 (1941).
[4] F. Braun, Electrician **57**, 222, 244 (1906).
[5] R. M. Foster, Bell Sys. Tech. J. **5**, 292 (1926).
[6] G. C. Southworth, Proc. I. R. E. **18**, 1502 (1930).

$\mathfrak{A}$ represents the amplitude of the normal velocity, and the angle of $\mathfrak{A}$ represents the phase of the vibration with respect to some arbitrary origin. It will be assumed that the normal vibrations are all of the same frequency. If this is not the case, the normal velocity at each point of the plane may be subjected to Fourier analysis; the amplitude $\mathfrak{A}$ is then to be equated to the amplitude of the component with the angular frequency ω.

The steady-state pressure at any point Q in the space above the plane is then given by

$$\mathcal{P}(Q)=i\omega\rho e^{i\omega t}\int\int\frac{\mathfrak{A}(x,y)}{2\pi l}e^{-ikl}dS, \qquad (1)$$

where ρ is the density of the medium, and l is the distance from the point x,y to the point Q.

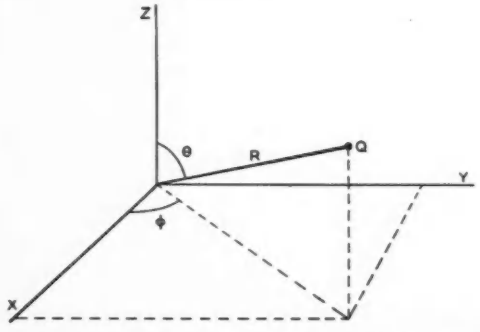

FIG. 1. The coordinate system used to specify the position of the field-point Q.

Consider now a spherical coordinate system with the same origin as the x,y,z system, and with the z axis as its polar axis. This spherical coordinate system may be completely defined by stating that the Cartesian coordinates of the point Q are

$$Q\equiv(R\sin\theta\cos\varphi,\ R\sin\theta\sin\varphi,\ R\cos\theta). \qquad (2)$$

See Fig. 1. It is evident that R is the distance from Q to the origin of coordinates, θ is the polar angle, and φ is the azimuth. The distance l is now given by

$$\begin{aligned}l^2&=(R\sin\theta\cos\varphi-x)^2\\&\quad+(R\sin\theta\sin\varphi-y)^2+(R\cos\theta)^2\\&=R^2+x^2+y^2-2R\sin\theta(x\cos\varphi+y\sin\varphi).\end{aligned} \qquad (3)$$

The expression (1) holds for any point above the plane, and is correct for any distribution $\mathfrak{A}(x,y)$ for which the integral is convergent. It is now desirable to introduce some restrictions which are equivalent to the assumption that Q lies in the Fraunhofer region of the pressure distribution. It is assumed (1) either that the normal velocity is zero outside a circle on the plane with its center at the origin and with radius r_0, or that the normal velocity decreases with increasing r in such a way that the surface outside a circle of radius r_0 makes no appreciable contribution to the pressure at Q; and (2) that R^2 is large* compared with r_0^2:

$$R^2\gg r_0^2. \qquad (4)$$

These assumptions make it permissible to approximate (3) by

$$l\cong R-\sin\theta(x\cos\varphi+y\sin\varphi). \qquad (5)$$

Subject to the condition (4), Eq. (1) now becomes

$$\mathcal{P}(R,\theta,\varphi)=\frac{i\omega\rho e^{i(\omega t-kR)}}{2\pi R}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathfrak{A}(x,y)\times e^{ik\sin\theta(x\cos\varphi+y\sin\varphi)}dxdy. \qquad (6)$$

In order to simplify the form of the equations which will appear in the remainder of this paper, the pressure $\mathcal{P}$ will be replaced by a quantity $\mathcal{Q}$ which differs from $\mathcal{P}$ by a multiplicative factor:

$$\mathcal{P}(R,\theta,\varphi)\equiv\frac{i\omega\rho e^{i(\omega t-kR)}}{2\pi R}\mathcal{Q}(\theta,\varphi), \qquad (7)$$

whence

$$\mathcal{Q}(\theta,\varphi)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathfrak{A}(x,y)e^{ik\sin\theta(x\cos\varphi+y\sin\varphi)}dxdy. \qquad (8)$$

Circular Symmetry

The very important special case will now be considered in which the distribution of normal velocity on the surface is circularly symmetric.

The result (8) will first be expressed in terms of polar coordinates in the plane. Let r be the

* The degree to which the strong inequality (4) must be satisfied is difficult to define in general, because it is possible to devise amplitude distributions of a special character such that the pattern at any pre-assigned R is greatly different from the pattern at infinity. Amplitude distributions which are useful in practice, however, such as those discussed in Part II, will not lead to any difficulty in this respect.

radius, and ψ the angle measured from the x to the y axis:

$$x=r\cos\psi, \quad y=r\sin\psi. \tag{9}$$

The Jacobian of the transformation from x,y to r, ψ is simply r. It therefore follows that

$$\mathcal{Q}(\theta, \varphi)=\int_0^\infty\int_0^{2\pi}\mathfrak{A}_p(r, \psi)e^{ikr\sin\theta\cos(\psi-\varphi)}rdrd\psi, \tag{10}$$

where

$$\mathfrak{A}_p(r, \psi)\equiv\mathfrak{A}(x, y). \tag{11}$$

The case of circular symmetry may now be considered, for in this case, the amplitude depends only on the radius r:

$$\mathfrak{A}_p(r, \psi)\rightarrow\mathfrak{A}_c(r). \tag{12}$$

Equation (10) then becomes

$$\mathcal{Q}(\theta)=\int_0^\infty\int_0^{2\pi}\mathfrak{A}_c(r)e^{ikr\sin\theta\cos(\psi-\varphi)}rdrd\psi \tag{13}$$

$$=2\pi\int_0^\infty\mathfrak{A}_c(r)J_0(kr\sin\theta)rdr, \tag{14}$$

where one of the integral representations[7] of J_0 has been used. Note that $\mathcal{Q}$ is now a function only of θ.

In the distributions which will be considered in the second part of this paper, the letter a will be used to denote a length which is characteristic of the distribution. The following two abbreviations will be used frequently:

$$\alpha\equiv\frac{2\pi a}{\lambda}\sin\theta\equiv ka\sin\theta$$
$$u\equiv r/a. \tag{15}$$

In terms of α and u, Eq. (14) becomes

$$Q(\alpha)=2\pi a^2\int_0^\infty A(u)J_0(\alpha u)udu, \tag{16}$$

where

$$Q(ka\sin\theta)\equiv\mathcal{Q}(\theta)$$

and

$$A(r/a)\equiv\mathfrak{A}_c(r). \tag{17}$$

A reciprocity exists between Q and A, since they are essentially Fourier transforms of one another. The result of a direct application of the Fourier-Bessel integral theorem[8] to (16) is

$$A(u)=\frac{1}{2\pi a^2}\int_0^\infty Q(\alpha)J_0(\alpha u)\alpha d\alpha. \tag{18}$$

This formula provides a means of calculating the velocity distribution which corresponds to any prescribed directional pattern.

By comparison of the form of Eqs. (16) and (18), it is evident that one might equally well consider $Q(u)$ as the distribution, with $4\pi^2a^4A(\alpha)$ as the corresponding directional pattern. The fact stated in the last sentence is of considerable theoretical interest, but for reasons to be given in Part II, it is of little practical value.

A similar reciprocity theorem may be established, in general, by application of the multiple Fourier integral theorem to Eq. (8). The exhibition of this theorem would require additional special notation, however, and for this reason it will not be given here.

Elliptical Symmetry

In the circular case just considered, the contours of equal amplitude are circles, with the origin of coordinates placed at their centers. This section is devoted to the consideration of the slightly more general case in which the contours of equal amplitude are similar (*not* confocal) ellipses with a common orientation and center. This generalization, although almost trivial from the mathematical point of view, is of some importance when one wishes to obtain a directional pattern which is more sharply peaked in the horizontal direction than in the vertical, or vice versa.

The coordinate axes are assumed to be oriented parallel to the principal axes of the ellipses, with the origin at their common center. The assumption of elliptical symmetry then means that the function $\mathfrak{A}_p(r, \psi)$ is a function of the single variable

$$\eta\equiv r(\epsilon\cos^2\psi+\epsilon^{-1}\sin^2\psi)^{\frac{1}{2}}, \tag{19}$$

where ϵ is the ratio of the length of the y axis of

[7] G. N. Watson, *A Treatise on the Theory of Bessel Functions* (Cambridge University Press, 1922), p. 20.

[8] E. T. Whittaker and G. N. Watson, *A Course of Modern Analysis* (Cambridge University Press, 1927), fourth edition, p. 385.

the ellipse to the length of the x axis. The function $\mathfrak{A}_p(r, \psi)$ thus becomes

$$\mathfrak{A}_p(r, \psi) \rightarrow \mathfrak{A}_e(\eta). \tag{20}$$

In addition to η, it is convenient to introduce also the new variable γ:

$$\tan \gamma \equiv \epsilon^{-1} \tan \psi. \tag{21}$$

The effect of the transformation from r, ψ to η, γ is to change the relative scale of the two Cartesian coordinate axes so that the ellipses are transformed into circles.

A simple calculation shows that the Jacobian determinant of the transformation is

$$\partial(r, \psi)/\partial(\eta, \gamma) = \eta/r, \tag{22}$$

and a somewhat longer calculation leads to the result that the expression $r \cos(\psi - \varphi)$, which appears in the exponent in (10), is given by

$$\begin{aligned} r \cos(\psi - \varphi) &= r(\cos \psi \cos \varphi + \sin \psi \sin \varphi) \\ &= \eta(\epsilon^{-\frac{1}{2}} \cos \gamma \cos \varphi \\ &\qquad + \epsilon^{\frac{1}{2}} \sin \gamma \sin \varphi) \qquad (23) \\ &= \eta[\epsilon^{-1} \cos^2 \varphi + \epsilon \sin^2 \varphi]^{\frac{1}{2}} \\ &\qquad \times \cos(\gamma - \tan^{-1} \epsilon \tan \varphi). \end{aligned}$$

Equation (10) thus becomes

$$\begin{aligned} \mathfrak{P}(\theta, \varphi) &= \int_0^\infty \int_0^{2\pi} \mathfrak{A}_e(\eta) \\ &\qquad \times \exp\{ik\eta \sin \theta[\epsilon^{-1} \cos^2 \varphi + \epsilon \sin^2 \varphi]^{\frac{1}{2}} \\ &\qquad \times \cos(\gamma - \tan^{-1} \epsilon \tan \varphi)\} \eta d\eta d\gamma \qquad (24) \\ &= 2\pi \int_0^\infty \mathfrak{A}_e(\eta) J_0(k\eta \sin \theta \\ &\qquad \times [\epsilon^{-1} \cos^2 \varphi + \epsilon \sin^2 \varphi]^{\frac{1}{2}}) \eta d\eta. \qquad (25) \end{aligned}$$

This result is capable of simple interpretation. Suppose that for a given distribution $\mathfrak{A}_c(r)$ in the circular case, the directionality function is $Q(ka \sin \theta)$. Then by comparing (25) with (14), it is evident that if $\mathfrak{A}_e$ is the same function of η that $\mathfrak{A}_c$ is of r, the directionality function which corresponds to $\mathfrak{A}_e(\eta)$ is

$$Q(ka \sin \theta[\epsilon^{-1} \cos^2 \varphi + \epsilon \sin^2 \varphi]^{\frac{1}{2}}). \tag{26}$$

The matter of elliptical symmetry will not be mentioned again in this paper. It is mentioned here because all of the numerical results which have been obtained for the circular case are directly applicable to the elliptical case by use of the correspondence principle enunciated in the preceding paragraph.

The Linear Case

The case of a continuous distribution of point radiators along a *line* will now be discussed. This special case is of considerable interest because of its simplicity.

The formulae for the linear case will be obtained by considering the linear distribution as a limit of a distribution on a surface. Consider the surface distribution in which the normal velocity is zero everywhere except upon a strip of width w along the x axis. Within this strip, the velocity is to be a function of x only. Define $\mathfrak{A}_l(x)$ by

$$\mathfrak{A}_l(x) \equiv w\mathfrak{A}(x, 0). \tag{27}$$

It is evident that $\mathfrak{A}_l(x)$ is equal to the amplitude of the normal velocity within the strip, multiplied by w.

It now follows directly from (1) and (27) that the pressure at any point Q above the surface is given by

$$\mathfrak{P}(Q) = i\omega\rho e^{i\omega t} \int \frac{\mathfrak{A}_l(x)}{2\pi l} e^{-ikl} dx, \tag{28}$$

where l is the distance from the point x on the x axis to the point Q, and where it has been assumed that w is very small compared with the wave-length. The distance l is given by

$$l^2 = (R \sin \theta - x)^2 + (R \cos \theta)^2. \tag{29}$$

The radius R has its previous meaning, but θ is now defined as the angle between the line from the origin to the point Q and the y,z plane, as shown in Fig. 2. The reason for using the same symbol for these two different angles will be apparent later.

On the basis of the same approximation as before, namely,

$$R^2 \gg x_0^2, \tag{30}$$

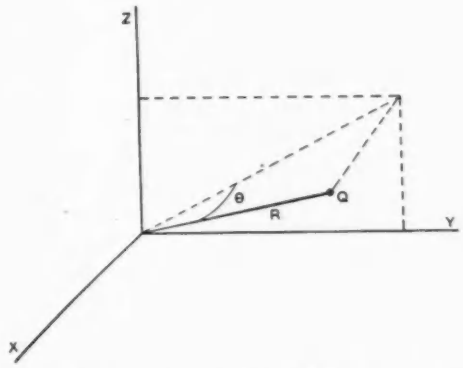

FIG. 2. The angle θ as used for the linear case only.

it follows that

$$l \cong R - x \sin \theta \tag{31}$$

and

$$\mathcal{Q}(\theta) = \int_{-\infty}^{\infty} \mathfrak{A}_l(x) e^{ikx \sin \theta} dx. \tag{32}$$

The directional pattern given by (28) or (32) is that corresponding to a linear source on the surface of an infinite baffle. If the baffle is not present, and if the width of the source is small compared with the wave-length, the directional pattern as given by (28) or (32) should be divided by two.

In the distributions which will be considered in the second part of this paper, the letter b will be used to denote a length which is characteristic of the distribution. The following abbreviations will be used frequently:

$$\beta = kb \sin \theta, \quad v = x/b. \tag{33}$$

In terms of β and v, Eq. (32) becomes

$$Q(\beta) = b \int_{-\infty}^{\infty} A(v) e^{i\beta v} dv, \tag{34}$$

where

$$Q(kb \sin \theta) \equiv \mathcal{Q}(\theta)$$

and

$$A(x/b) \equiv \mathfrak{A}_l(x). \tag{35}$$

Just as in the circular case, a reciprocity exists between Q and A. The result of a direct application of the Fourier integral theorem* to (34) is

$$A(v) = \frac{1}{2\pi b} \int_{-\infty}^{\infty} Q(\beta) e^{-i\beta v} d\beta. \tag{36}$$

* Reference 8, p. 189.

By comparison of the form of Eqs. (34) and (36), it is clear that one might equally well consider $Q(v)$ as the distribution, and $2\pi b^2 A(-\beta)$ as the corresponding directional pattern.

The Relation Between Circular and Linear Distributions

The relation which will be considered is that between the linear distribution, and the surface distribution when the point Q lies in the x,z plane. It follows from (8) that when $\varphi = O$

$$\mathcal{Q}(\theta, 0) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathfrak{A}(x,y) e^{ikx \sin \theta} dx dy, \tag{37}$$

and it should be noted that the angles θ which occur in (32) and (37) and which are not equal in general, are equal when $\varphi = 0$. Accordingly, in this section it will not be necessary to differentiate between the two θ's.

By comparison of (32) with (37), it is now evident that in order to obtain in the plane $\varphi = 0$ the same directional pattern in the linear case that one obtains with a surface distribution, it is sufficient to have

$$\mathfrak{A}_l(x) = \int_{-\infty}^{\infty} \mathfrak{A}(x, y) dy. \tag{38}$$

If, furthermore, the patterns are to be the same for all values of k—that is to say, for all frequencies of vibration—then it is also necessary that the relation between $\mathfrak{A}_l(x)$ and $\mathfrak{A}(x, y)$ be as specified by Eq. (38).

When further the surface distribution is spherically symmetrical, Eq. (38) may be written

$$\mathfrak{A}_l(x) = \int_{-\infty}^{\infty} \mathfrak{A}_c[(x^2+y^2)^{\frac{1}{2}}] dy \tag{39}$$

or

$$\mathfrak{A}_l(x) = 2 \int_{x}^{\infty} \mathfrak{A}_c(r) \frac{r dr}{(r^2-x^2)^{\frac{1}{2}}}. \tag{40}$$

Thus if the circular distribution is given, an integration suffices to determine the corresponding linear distribution.

The converse problem is also of interest: given $\mathfrak{A}_l(x)$, to find $\mathfrak{A}_c(r)$. This problem is more difficult. The solution may be found by using sub-

stitutions which convert (40) to Abel's integral equation.* The use of the substitutions

$$\xi=1/x^2, \quad \rho=1/r^2 \tag{41}$$

leads to

$$[\xi^{-\frac{1}{2}}\mathfrak{a}_l(\xi^{-\frac{1}{2}})]=\int_0^{\xi}[\rho^{-\frac{1}{2}}\mathfrak{a}_c(\rho^{-\frac{1}{2}})]\frac{d\rho}{(\xi-\rho)^{\frac{1}{2}}}. \tag{42}$$

This integral equation is now in Abel's canonical form; its solution is therefore

$$[\rho^{-\frac{1}{2}}\mathfrak{a}_c(\rho^{-\frac{1}{2}})]=\frac{1}{\pi}\frac{d}{d\rho}\int_0^{\rho}[\xi^{-\frac{1}{2}}\mathfrak{a}_l(\xi^{-\frac{1}{2}})]\frac{d\xi}{(\rho-\xi)^{\frac{1}{2}}}, \tag{43}$$

or

$$\mathfrak{a}_c(r)=-\frac{1}{\pi}\frac{d}{dr}\int_r^{\infty}\mathfrak{a}_l(x)\frac{rdx}{x(x^2-r^2)^{\frac{1}{2}}}. \tag{44}$$

Leibniz' rule** is not applicable to this integral, since the integrand becomes infinite at the lower limit.

PART II. NUMERICAL RESULTS

In Part I the general theory of surface distributions of sources has been examined briefly. In this part the directional patterns of a number of specific source distributions will be determined.

It was shown in Part I [Eqs. (18) and (36)] that instead of prescribing the velocity distribution and then determining the directional pattern, it is just as easy to prescribe a desired directional pattern, and then to determine the distribution which would yield this pattern. Unfortunately, however, it is not very practical to proceed in this manner, because only certain very special directional patterns correspond to sources of finite extent. In practice, the sources are restricted to some given size, and the aim is to obtain with a source of the given size the directional pattern which is best suited to the requirements. Accordingly, the procedure will be to obtain, in the form of tabulated functions, the patterns corresponding to a number of suitably chosen distributions. By simple superposition of these patterns, one may obtain a wide variety of patterns; the corresponding velocity distributions may then also be obtained by superposition.

The discussion will be limited to linear distributions, and to distributions with circular symmetry. In all of the cases to be treated, it will be assumed that the normal velocity has everywhere the same phase, so that the velocity distribution functions may be considered to be real. Furthermore, in the linear case, only those distributions which are symmetrical with respect to the origin will be considered. Accordingly, all of the directionality functions will be real.

The two fundamental formulae which will be used in this part are

$$Q(\alpha)=2\pi a^2\int_0^{\infty}A(u)J_0(\alpha u)udu \tag{16}$$

and

$$Q(\beta)=2b\int_0^{\infty}A(v)\cos\beta v dv, \tag{45}$$

where (45) is obtained from (34) by use of the fact that $A(v)$ is to be an even function of its argument. The abbreviations α, β, u, and v are defined in (15) and (33).

It is convenient to express A in such a form that it is equal to unit velocity when its argument is zero. It is also convenient to denote by Q_N an expression proportional to Q which is equal to unity when its argument is zero:

$$Q_N(\alpha)=Q(\alpha)/N, \quad Q_N(\beta)=Q(\beta)/N, \tag{46}$$

where

$$N\equiv Q(0)=2\pi a^2\int_0^{\infty}A(u)udu$$

or

$$N\equiv Q(0)=2b\int_0^{\infty}A(v)dv. \tag{47}$$

Distributions of Infinite Extent

Before discussing the distributions of finite extent, three specific distributions will be examined which are not limited to a finite region. Although these distributions cannot be realized in practice, they are nevertheless interesting and important, because in some respects they represent limiting cases of what may be done with distributions limited to a finite region.

The Gaussian distribution is probably the most interesting distribution of infinite extent.

* Reference 8, p. 229.
** Reference 8, p. 67.

In the circular case, this distribution is of the form

$$A(r/a)=A_0\exp\left[-(r/a)^2\right],$$

where A_0 is a constant with the dimensions of a velocity. In the remainder of this paper, the constant A_0 will be omitted for the sake of simplicity. The patterns for the Gaussian distribution are given below*

$$\begin{aligned}A(r/a)&=\exp(-r^2/a^2)=\exp(-u^2),\\ A(x/b)&=\exp(-x^2/b^2)=\exp(-v^2).\end{aligned}\tag{48}$$

$$\begin{aligned}Q(\alpha)&=2\pi a^2\int_0^\infty \exp(-u^2)J_0(\alpha u)u\,du\\ &=\pi a^2\exp(-\alpha^2/4),\\ Q_N(\alpha)&=\exp(-\alpha^2/4).\end{aligned}\tag{49}$$

$$\begin{aligned}Q(\beta)&=2b\int_0^\infty \exp(-v^2)\cos\beta v\,dv\\ &=\pi^{\frac{1}{2}}b\exp(-\beta^2/4),\\ Q_N(\beta)&=\exp(-\beta^2/4).\end{aligned}\tag{50}$$

It is evident that the pattern is also Gaussian in form.

The Gaussian distribution is particularly interesting because both the distribution and the pattern approach zero very rapidly as the argument increases from zero. It therefore offers the possibility of being approximated rather closely by a distribution of finite extent.

Another distribution of some interest is algebraic in form**

$$A(u)=1/(1+u^2)^{\mu+1},\quad A(v)=1/(1+v^2)^{\mu+\frac{1}{2}}.\tag{51}$$

$$\begin{aligned}Q(\alpha)&=2\pi a^2\int_0^\infty \frac{J_0(\alpha u)u\,du}{(1+u^2)^{\mu+1}}\\ &=(\pi a^2/\mu)M_\mu(|\alpha|),\\ Q_N(\alpha)&=M_\mu(|\alpha|).\end{aligned}\tag{52}$$

$$\begin{aligned}Q(\beta)&=2b\int_0^\infty \frac{\cos\beta v\,dv}{(1+v^2)^{\mu+\frac{1}{2}}}\\ &=2f_{\mu-1}bM_\mu(|\beta|),\\ Q_N(\beta)&=M_\mu(|\beta|).\end{aligned}\tag{53}$$

In order to secure convergence of the normalization integrals (46) and (47), it is necessary to postulate that μ be greater than zero. The number f is discussed later on [cf. Eqs. (84) and (85)].

The function M_μ is defined by

$$M_\mu(z)\equiv z^\mu K_\mu(z)/2^{\mu-1}\Gamma(\mu),\tag{54}$$

where K_μ is a Bessel function of the second kind for imaginary argument.* The function K_μ is always positive for positive values of z, as may be seen from one of its integral representations**

$$K_\mu(z)=\int_0^\infty e^{-z\cosh t}\cosh\mu t\,dt,\quad z>0.\tag{55}$$

Furthermore, M_μ decreases monotonically with z, when z is positive†

$$\frac{d}{dz}z^\mu K_\mu(z)=-z^\mu K_{\mu-1}(z).\tag{56}$$

When μ is half-integral, M_μ may be expressed in a simple manner††

$$M_{\frac{1}{2}}(z)=e^{-z},\tag{57}$$

$$M_{n+\frac{1}{2}}(z)=e^{-z}\left(1+z+\frac{2n-2}{2n-1}\frac{z^2}{2!}+\frac{(2n-2)(2n-4)}{(2n-1)(2n-2)}\frac{z^3}{3!}+\cdots+\frac{2^n n!}{(2n)!}z^n\right).\tag{58}$$

It is worth noting that in the special case $\mu=\frac{1}{2}$, Eqs. (52) and (53) may be written

$$Q_N(\alpha)=e^{-|\alpha|},\quad Q_N(\beta)=e^{-|\beta|}.\tag{59}$$

The third and last class of distributions of infinite extent to be considered here may be regarded as the inverse of the case just treated***

$$A(u)=M_\mu(u),\quad A(v)=M_\mu(|v|).\tag{60}$$

$$\begin{aligned}Q(\alpha)&=2\pi a^2\int_0^\infty M_\mu(u)J_0(\alpha u)u\,du\\ &=4\pi\mu a^2/(1+\alpha^2)^{\mu+1}\\ Q_N(\alpha)&=1/(1+\alpha^2)^{\mu+1}.\end{aligned}\tag{61}$$

* Reference 7, p. 393.
** Reference 7, pp. 434 and 79, 172.
* Reference 7, p. 78.
** Reference 7, p. 181.
† Reference 7, p. 79.
†† Reference 7, p. 80.
*** Reference 7, p. 410.

$$Q(\beta)=2b\int_0^\infty M_\mu(v)\cos\beta v dv$$
$$=\frac{\pi b}{f_{\mu-1}}\frac{1}{(1+\beta^2)^{\mu+\frac{1}{2}}}, \tag{62}$$
$$Q_N(\beta)=1/(1+\beta^2)^{\mu+\frac{1}{2}}.$$

It is again necessary to postulate that μ be greater than zero.

The results (61) and (62) might have been obtained directly from (52) and (53) by application of the reciprocity theorem stated in the paragraphs following Eqs. (18) and (36).

Definition of Efficiency

The distributions of infinite extent which have just been discussed are all characterized by the fact that the average amplitude of the vibrating surface is zero. At the other extreme is the circular piston, over which the average amplitude is equal to the maximum amplitude. In nearly all of the sources of finite size which we are about to discuss, the average amplitude of the vibrating surface will be less than the maximum amplitude, and the word "efficiency" will be used to denote the ratio of the average amplitude to the maximum amplitude.

The efficiency is evidently given by

$$E=2\int_0^{u_{\max}} A(u)udu/(A_{\max}u_{\max}^2) \tag{63}$$

or

$$E=\int_0^{v_{\max}} A(v)dv/(A_{\max}v_{\max}). \tag{64}$$

It follows from (18) and (36) that one may obtain any directional pattern whatever if one is not interested in efficiency. In general, one finds that the greater the efficiency, the less the freedom in adjusting the form of the directional pattern.

Distributions of Finite Extent

In this section the directional patterns of distributions restricted to a finite region will be examined. Accordingly, the problem is to evaluate the integrals of the form (16) and (45) in which the upper limit of the integral is finite.

For the linear case, as represented by Eq. (45), a large number of distributions are known for which the integral can be determined analytically; a few of these distributions will be examined numerically later on in this section. In the circular case, however, only a few finite integrals of the form (16) are known.

Of these, the most useful integral is that in which $A(u)$ is an integral power of u. By repeated use of a reduction formula,* the integral

$$\int_0^1 u^n J_0(\alpha u)udu$$

may be expressed in terms of J_0 and J_1 when n is even, and in terms of Bessel *and* Struve functions of the zero and first order when n is odd. The corresponding integral in the linear case may also be treated by use of a reduction formula, and the result may be expressed in terms of trigonometric functions. Unfortunately, however, the results obtained in this manner turn out to involve heavy cancellation between terms, so that the results are not well adapted either to ready interpretation or to computation. For these reasons, the procedure will run along somewhat different lines.

The numerical treatment of the circular case will be limited to distributions of the form

$$A(u)=(1-u^{2m})^n, \tag{65}$$

where m and n are integers. It is evident that every distribution of this form is a polynomial in u^2. It might at first be thought that this distribution would be more difficult to handle than a simple power of u, but this is not the case.

The result of substituting (65) in (16) is

$$Q(\alpha)=2\pi a^2\int_0^1 (1-u^{2m})^n J_0(\alpha u)udu \tag{66}$$
$$=2\pi a^2\int_0^{\pi/2} J_0(\alpha\sin\theta)(1-\sin^{2m}\theta)^n \times\sin\theta\cos\theta d\theta, \tag{67}$$

where

$$u=\sin\theta. \tag{68}$$

* Reference 7, pp. 132, 133, 329, and 752.

This integral will be evaluated by making use of the relation*

$$\int_0^{\pi/2} J_0(\alpha \sin\theta)\cos^{2n-1}\theta \sin\theta d\theta = \frac{G_n}{2n}, \quad (69)$$

where

$$G_n = 2^n n! J_n(\alpha)/\alpha^n$$
$$= 1 - \frac{\alpha^2}{2^2\cdot 1\cdot(n+1)} + \frac{\alpha^4}{2^4\cdot 1\cdot 2\cdot(n+1)(n+2)} - \cdots \quad (70)$$

is a Bessel function normalized so that it approaches unity as α approaches zero. By use of two successive binomial expansions one finds

$$(1-\sin^{2m}\theta)^n = (1-(1-\cos^2\theta)^m)^n \quad (71)$$
$$= \sum_{k=0}^{n}\sum_{l=0}^{mk} C_k^n C_l^{mk}(-)^{k+l}\cos^{2l}\theta \quad (72)$$
$$= \sum_{l=0}^{mn} \Omega_{lmn}\cos^{2l}\theta, \quad (73)$$

where

$$\Omega_{lmn} = \sum_{k=\lambda}^{n} C_k^n C_l^{mk}(-1)^{k+l}, \quad (74)$$

and where λ is the smallest integer which is greater than or equal to l/m. Equation (73) was obtained from (72) by reversing the order of summation. It is possible that the sum indicated in (74) can be performed in general. No great effort has been made to perform this summation, however, because in practice one is interested only in small values of m and n, in which case the evaluation of the Ω's offers no problem.

From (67), (69), and (73), the final expression for $Q(\alpha)$ is

$$Q(\alpha) = \pi a^2 \sum_{l=0}^{mn} \Omega_{lmn}\frac{G_{l+1}}{l+1}. \quad (75)$$

By (63), the efficiency is given by

$$E = 2\int_0^1 (1-u^{2m})^n u du \quad (76)$$
$$= m^n n!/(1+m)(1+2m)\cdots(1+nm), \quad (77)$$

and the normalization constant is obtained by use of the same integral:

$$N = \pi a^2 E. \quad (78)$$

The special cases of $m=1$ and $n=1$ are of particular interest:

$$\left.\begin{aligned} Q_N(\alpha) &= G_{n+1} \\ E &= 1/(n+1) \end{aligned}\right\} m=1 \quad (79)$$

$$\left.\begin{aligned} Q_N(\alpha) &= \frac{1}{m}\sum_{l=2}^{m+1} C_l^{m+1}(-1)^l G_l \\ E &= m/(m+1). \end{aligned}\right\} n=1. \quad (80)$$

The treatment of the linear case with the distribution

$$A(v) = (1-v^{2m})^n \quad (81)$$

is very similar, in view of the relation*

$$\int_0^{\pi/2} \cos(\beta\sin\theta)\cos^{2n+1}\theta d\theta = f_n H_n, \quad (82)$$

where

$$H_n \equiv \left(\frac{\pi}{2\beta}\right)^{\frac{1}{2}} 1\cdot 3\cdot 5\cdots(2n+1)\cdot\frac{J_{n+\frac{1}{2}}(\beta)}{\beta^n}$$
$$= 1 - \frac{\beta^2}{2\cdot 1\cdot(2n+3)} + \frac{\beta^4}{2^2\cdot 1\cdot 2\cdot(2n+3)(2n+5)} - \cdots \quad (83)$$

is a normalized Bessel function of half-integral order.

The number f_n is defined by

$$f_n \equiv \int_0^{\pi/2} \sin^{2n+1}\theta d\theta = \frac{2\cdot 4\cdots(2n)}{3\cdot 5\cdots(2n+1)} \quad (84)$$

and a second approximation to its value for large values of n, obtained by the method of steepest descents, is

$$f_n \leqq \left(\frac{\pi}{4n+3}\right)^{\frac{1}{2}}. \quad (85)$$

The writer is indebted to Dr. R. M. Foster for

* Reference 7, p. 372.

* Reference 7, p. 48.

pointing out that the approximate formula (85) is surprisingly accurate even for small values of n. For $n=2$, the approximate expression is only 0.2 percent too large, for $n=1$, 0.5 percent, and even for $n=0$, the error is only 2.3 percent.

One obtains

$$Q(\beta)=2b\sum_{l=0}^{mn}\Omega_{lmn}f_lH_l \tag{86}$$

$$E=(2m)^n n!/(1+2m)(1+4m)\cdots(1+2mn) \tag{87}$$

$$N=2bE \tag{88}$$

and in the special cases $m=1$ and $n=1$, one has

$$\left.\begin{aligned}Q_N(\beta)&=H_n\\E&=f_n\end{aligned}\right\}m=1. \tag{89}$$

$$\left.\begin{aligned}Q_N(\beta)&=\frac{2m+1}{2m}\sum_{l=1}^{m}C_l^m(-1)^{l+1}f_lH_l\\E&=\frac{2m}{2m+1}.\end{aligned}\right\}n=1. \tag{90}$$

In Tables I to XIX are tabulated the directionality functions for 10 different choices of m and n:

m	n
1	−1
—	0
1	1
1	2
1	3
1	4
2	1
2	2
3	1
4	1.

Tables I to X are for the circular case, and Tables XI to XIX for the linear case. In each table Q_N is tabulated for values of α or β lying in the range zero to twenty, with an interval of 0.4. The square of the directionality function, as well as ten times the logarithm of its inverse square, is also tabulated.

For the sake of completeness, the directional pattern of a circular ring is tabulated in Table I. The circular ring corresponds formally to $m=1$, $n=-1$, and is so indicated in the above list. The distribution which corresponds to the circular ring in the linear case is two point sources, separated by a distance $2b$, and this distribution also corresponds formally to $m=1$, $m=-1$. The corresponding pattern, $H_{-1}=\cos\beta$, is not tabulated.

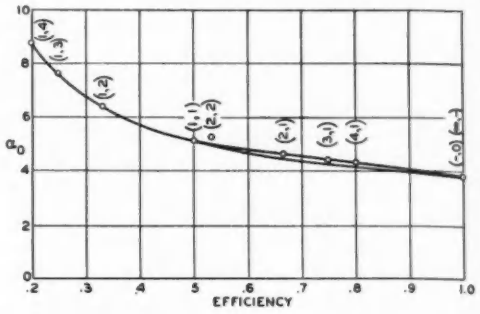

FIG. 3. The position of the first node for the circular case.

Table II, which contains the directional pattern of a circular piston, is more detailed than the other tables, insofar as the pattern and its square are tabulated to six significant figures, with an interval of 0.1. This table represents a slight extension and correction of a similar table given by E. Lommel.[9]

In Table XX is collected some information which is of interest in comparing the directionality patterns for the various sources. The columns labeled α_0 and β_0 are the values of α and β which correspond to the first null of the pattern. The columns to the right of these columns state the number of decibels by which the amplitude of the first minor lobe of the pattern is below that of the principal lobe. The last column in each of the sections gives the efficiency of the pattern.

In general, one would expect that the patterns of low efficiency would correspond to greater suppressions of the minor lobes, and to greater spreads of the principal lobe. This tendency is indicated in Table XX, and is exhibited graphically in Figs. 3 to 6. In these figures, the position of the first null and the degree of suppression of the first minor lobe are plotted as functions of the efficiency.

It has been stated already that a large number of patterns may be studied analytically in the

[9] E. Lommel, Abhand. d. Bayer. Akad. d. Wiss. Math.-phys. Klasse **15**, 229–328 (1886).

linear case. In addition to those treated in Tables XI to XIX, numerical results will be given for only two others. The first of these is

$$A(v)=\cos(2v/\pi), \tag{91}$$

$$Q(\beta)=\frac{4b}{\pi}\frac{\cos\beta}{1-(2\beta/\pi)^2}, \tag{92}$$

$$Q_N(\beta)=\frac{\cos\beta}{1-(2\beta/\pi)^2}, \tag{93}$$

$$E=2/\pi=0.6366. \tag{94}$$

This pattern is tabulated in Table XXI. The first null occurs at $\beta=4.7$, and the first minor

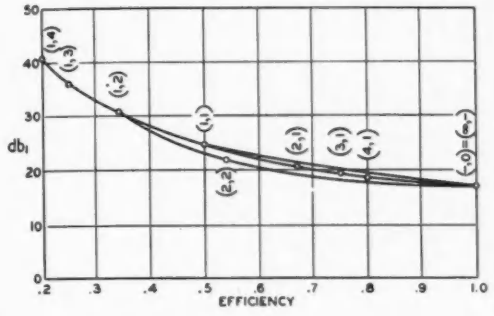

FIG. 4. The magnitude of the first minor lobe for the circular case.

lobe is 23 db down. The second is

$$A(v)=\cos^2(2v/\pi), \tag{95}$$

$$Q(\beta)=\frac{b}{1-(\beta/\pi)^2}\frac{\sin\beta}{\beta}, \tag{96}$$

$$Q_N(\beta)=\frac{\sin\beta}{\beta(1-(\beta/\pi)^2)}, \tag{97}$$

$$E=\tfrac{1}{2}. \tag{98}$$

The first null occurs at $\beta=6.3$, and the first minor lobe is 31.5 db down. These two distributions are indicated by crosses in Figs. 5 and 6.

Omitted here are the triangular and trapezoidal distributions which are treated fully by Menges.[4] The normalized pattern of the triangular distribution is just $H_0^2(\beta/2)$, and the trapezoidal distribution may be obtained by superposing a

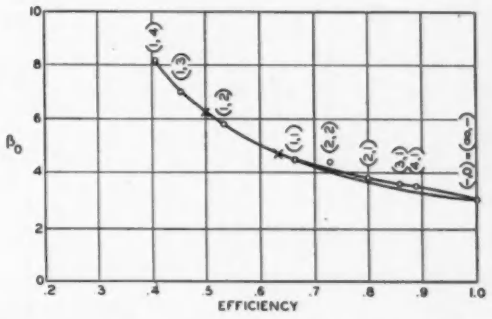

FIG. 5. The position of the first node for the linear case.

triangle and a rectangle in the manner described in the next section.

Superposition of Distributions

The usefulness of the results which have been obtained in the last section lies to a considerable degree in the possibility of combining them. If, for example, a given distribution $A(u)$ may be expressed as the algebraic sum of a number of other distributions $A_1(u)$, $A_2(u)$, $\cdots$:

$$A(u)=\Sigma_k B_k A_k(u), \tag{99}$$

where the B_k are numerical coefficients, and where the A_k are normalized so that $A_k(0)=1$, then the normalized directional pattern corresponding to $A(u)$ is

$$Q_N(\alpha)=\frac{\Sigma_k B_k E_k Q_{Nk}(\alpha)}{\Sigma_k B_k E_k}, \tag{100}$$

where Q_{Nk} is the normalized pattern corresponding to $A_k(u)$, and where E_k is the value of the efficiency which corresponds to $A_k(u)$. Con-

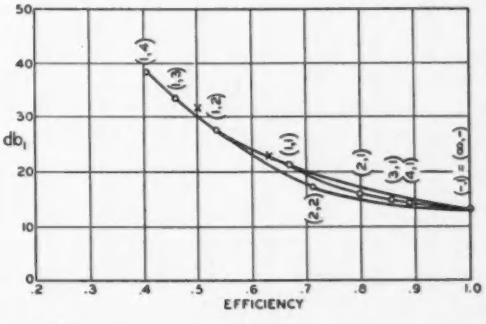

FIG. 6. The magnitude of the first minor lobe for the linear case.

versely, if

$$Q_N(\alpha)=\Sigma_k C_k Q_{Nk}(\alpha), \tag{101}$$

then

$$A(u)=\frac{\Sigma_k C_k A_k(u)/E_k}{\Sigma_k C_k/E_k}. \tag{102}$$

Equations (99) to (102) are valid for the circular case only; they hold for the linear case if α is replaced by β, and u by v. These results may easily be generalized to include the case in which the component distributions correspond to different values of a or of b, but the results would be awkward to express in the notation used, since α and u, and β and v, are defined with respect to a given value of a or b.

It would be very pleasant if it were possible to express any function of z in the forms

$$f(z)=\sum_k C_k G_k(z) \tag{103}$$

or

$$f(z)=\Sigma_k C_k H_k(z), \tag{104}$$

since if this were possible one could compute at once the circular or linear distribution *which would be restricted to a finite domain*, and which would correspond to the pattern $f(z)$. Even if such expansions were possible, however, it would be difficult to determine the coefficients C_k, because neither the G_k's nor the H_k's form an orthogonal set.

Expansions of the form (103) or (104) are not possible in general because the basis functions do not form a complete set. In practice, however, there is the great advantage that the expansion need be approximately valid only for values of α or β not greater than ka or kb, since only such values correspond to realizable values of θ. It is therefore possible to determine the optimum values of the C_k's in any given case by a least squares procedure, or by requiring the directionality function to have specified values at discrete values of α or β.

No very complete examination of the usefulness of the methods just described has yet been made, but the writer considers it possible that the use of linear combinations of the G_k's or of the H_k's may provide an expeditious method of determining the distributions which correspond to directional patterns of specified properties.

ACKNOWLEDGMENT

The writer is heavily indebted to Dr. R. M. Foster for checking all of the equations in this paper, and he wishes to thank Dr. L. A. MacColl for assistance in solving the integral equation (40).

PART III. TABLES

The nature of these tables, as well as the notation used, is described fully in Part II. For the sake of convenience in use, a brief description of the tables will also be given here.

TABLE I. Circular case: $m=1$, $n=-1$.
a = radius of circular ring.
$Q_N=G_0=J_0$.

α	Q_N	Q_N^2	$10\log_{10}Q_N^{-2}$
0.0	1.00000	1.00000	0.0
0.4	.96040	.92236	0.3511
0.8	.84629	.71620	1.450
1.2	.67113	.45042	3.464
1.6	.45540	.20739	6.832
2.0	.22389	.050127	13.00
2.4	+.0025077	.(5)62885	52.02
2.8	−.18504	.034238	14.65
3.2	−.32019	.102520	9.892
3.6	−.39177	.15348	8.139
4.0	−.39715	.15773	8.021
4.4	−.34226	.11714	9.313
4.8	−.24043	.057804	12.38
5.2	−.11029	.012164	19.15
5.6	+.026971	.00072743	31.38
6.0	.15065	.022694	16.44
6.4	.24331	.059200	12.28
6.8	.29310	.085905	10.66
7.2	.29507	.087067	10.60
7.6	.25160	.063304	11.99
8.0	.17165	.029464	15.31
8.4	+.069157	.0047827	23.20
8.8	−.039234	.0015393	28.13
9.2	−.13675	.018700	17.28
9.6	−.20898	.043672	13.60
10.0	−.24594	.060485	12.18
10.4	−.24337	.059230	12.27
10.8	−.20320	.041291	13.84
11.2	−.13299	.017687	17.52
11.6	−.044616	.0019906	27.01
12.0	+.047689	.0022743	26.43
12.4	.12956	.016786	17.75
12.8	.18870	.035608	14.48
13.2	.21669	.046953	13.28
13.6	.21013	.044156	13.55
14.0	.17107	.029266	15.34
14.4	.10648	.011339	19.45
14.8	+.027082	.(3)73345	31.35
15.2	−.054421	.0029616	25.28
15.6	−.12533	.015707	18.04

TABLE I.—*Continued.*

α	Q_N	Q_N^2	$10 \log_{10} Q_N^{-2}$
16.0	−.17490	.030590	15.14
16.4	−.19597	.038406	14.16
16.8	−.18597	.034586	14.61
17.2	−.14719	.021665	16.64
17.6	−.086328	.0074525	21.28
18.0	−.013356	.00017838	37.49
18.4	+.060098	.0036118	24.42
18.8	.12259	.015027	18.23
19.2	.16461	.027095	15.67
19.6	.18004	.032415	14.89
20.0	.16702	.027897	15.54
	Maxima of Q_N^2		
3.832	−.40276	.16221	7.899
7.016	+.30012	.090070	10.45
10.173	−.24970	.062353	12.05
13.324	+.21836	.047681	13.22
16.471	−.19646	.038598	14.13
19.616	+.18006	.032422	14.89
22.760	−.16718	.027951	15.54
25.904	+.15673	.024563	16.10
29.047	−.14801	.021908	16.59
32.190	+.14061	.019770	17.04
35.332	−.13421	.018013	17.44
38.475	+.12862	.016542	17.81

TABLE II. Circular case: $n=0$.
$A(u)=1,\ u \leqq 1.$
$A(u)=0,\ u>1.$
$Q_N=G_1=2J_1/\alpha.$

α	Q_N	Q_N^2	$10 \log_{10} Q_N^{-2}$
0.0	1.000000	1.000000	0.0
0.1	.998751	.997503	0.010844
0.2	.995008	.990042	0.04347
0.3	.988792	.977710	0.09791
0.4	.980133	.960661	0.1743
0.5	.969074	.939104	0.2728
0.6	.955670	.913305	0.3938
0.7	.939988	.883577	0.5377
0.8	.922105	.850278	0.7044
0.9	.902110	.813803	0.8948
1.0	.880101	.774578	1.109
1.1	.856186	.733055	1.349
1.2	.830482	.689700	1.613
1.3	.803113	.644990	1.904
1.4	.774211	.599403	2.223
1.5	.743915	.553410	2.570
1.6	.712370	.507471	2.946
1.7	.679724	.462024	3.353
1.8	.646130	.417484	3.794
1.9	.611744	.374231	4.269
2.0	.576725	.332611	4.781
2.1	.541231	.292931	5.332
2.2	.505421	.255450	5.927
2.3	.469454	.220387	6.568
2.4	.433488	.187912	7.260
2.5	.397675	.158146	8.009
2.6	.362168	.131166	8.822
2.7	.327112	.1070023	9.706
2.8	.292649	.0856437	10.673
2.9	.258916	.0670372	11.737
3.0	.226039	.0510938	12.916
3.1	.194143	.0376914	14.238
3.2	.163340	.0266798	15.738
3.3	.133735	.0178852	17.475
3.4	.105427	.0111148	19.541
3.5	.0785014	.00616248	22.102
3.6	.0530364	.00281286	25.509
3.7	.0290995	.000846778	30.722
3.8	+.00674790	.(4)455341	43.417
3.9	−.0139713	.000195197	37.095
4.0	−.0330217	.001090430	29.624
4.1	−.0503772	.00253786	25.955
4.2	−.0660224	.00435895	23.606
4.3	−.0799519	.00639230	21.943
4.4	−.0921707	.00849544	20.708
4.5	−.1026935	.01054595	19.769
4.6	−.111545	.0124422	19.051
4.7	−.118758	.0141034	18.507
4.8	−.124375	.0154691	18.105
4.9	−.128447	.0164986	17.826
5.0	−.131032	.0171693	17.652
5.1	−.132195	.0174755	17.576
5.2	−.132009	.0174263	17.588
5.3	−.130551	.0170436	17.684
5.4	−.127905	.0163598	17.862
5.5	−.124159	.0154155	18.120
5.6	−.119405	.0142574	18.460
5.7	−.113736	.0129359	18.882
5.8	−.107251	.0115028	19.392
5.9	−.100048	.0100097	19.996
6.0	−.0922280	.00850599	20.703
6.1	−.0838901	.00703755	21.526
6.2	−.0751344	.00564517	22.483
6.3	−.0660593	.00436384	23.601
6.4	−.0567617	.00322189	24.919
6.5	−.0473358	.00224068	26.496
6.6	−.0378728	.00143435	28.433
6.7	−.0284603	.000809990	30.915
6.8	−.0191820	.000367947	34.342
6.9	−.0101165	.0001023446	39.899
7.0	−.00133795	.(5)179011	57.471
7.1	+.00708543	.(4)502033	42.993
7.2	.0150910	.000227737	36.426
7.3	.0226220	.000511756	32.909
7.4	.0296284	.000877842	30.566
7.5	.0360662	.00130077	28.858
7.6	.0418984	.00175547	27.556
7.7	.0470942	.00221786	26.541
7.8	.0516300	.00266565	25.742
7.9	.0554885	.00307897	25.116

TABLE II.—*Continued.*

α	Q_N	Q_N^2	$10 \log_{10} Q_N^{-2}$
8.0	.0586591	.00344089	24.633
8.1	.0611377	.00373782	24.274
8.2	.0629265	.00395974	24.023
8.3	.0640336	.00410030	23.872
8.4	.0644729	.00415676	23.812
8.5	.0642640	.00412986	23.841
8.6	.0634314	.00402354	23.954
8.7	.0620044	.00384454	24.152
8.8	.0600167	.00360201	24.435
8.9	.0575062	.00330696	24.806
9.0	.0545137	.00297175	25.270
9.1	.0510837	.00260954	25.834
9.2	.0472628	.00223377	26.510
9.3	.0430998	.00185759	27.310
9.4	.0386451	.00149345	28.258
9.5	.0339504	.00115263	29.383
9.6	.0290677	.000844929	30.732
9.7	.0240492	.000578364	32.378
9.8	.0189470	.000358987	34.449
9.9	.0138121	.000190774	37.195
10.0	.00869455	.(4)755952	41.215
10.1	+.00364268	.(4)132691	48.772
10.2	−.00129720	.(5)168274	57.740
10.3	−.00608113	.(4)369802	44.320
10.4	−.01066784	.000113803	39.438
10.5	−.0150191	.000225572	36.467
10.6	−.0190997	.000364800	34.379
10.7	−.0228784	.000523421	32.811
10.8	−.0263271	.000693118	31.592
10.9	−.0294220	.000865652	30.627
11.0	−.0321428	.001033158	29.858
11.1	−.0344736	.00118843	29.250
11.2	−.0364023	.00132513	28.777
11.3	−.0379212	.00143802	28.422
11.4	−.0390264	.00152306	28.173
11.5	−.0397180	.00157752	28.020
11.6	−.0400001	.00160001	27.959
11.7	−.0398804	.00159044	27.985
11.8	−.0393703	.00155002	28.097
11.9	−.0384846	.00148106	28.294
12.0	−.0372412	.00138691	28.580
12.1	−.0356610	.00127171	28.956
12.2	−.0337675	.00114025	29.430
12.3	−.0315868	.000997726	30.010
12.4	−.0291468	.000849537	30.708
12.5	−.0264774	.000701053	31.542
12.6	−.0236099	.000557427	32.538
12.7	−.0205767	.000423401	33.732
12.8	−.0174112	.000303149	35.183
12.9	−.0141470	.000200138	36.987
13.0	−.01081816	.000117033	39.317
13.1	−.00745839	.(4)556276	42.547
13.2	−.00410102	.(4)168183	47.742
13.3	−.000778569	.(6)606169	62.174
13.4	+.00247747	.(5)613784	52.120
13.5	.00563693	.(4)317750	44.979
13.6	.00867126	.(4)751907	41.239
13.7	.0115537	.000133488	38.746
13.8	.0142595	.000203333	36.918
13.9	.0167662	.000281104	35.511
14.0	.0190536	.000363039	34.400
14.1	.0211042	.000445386	33.513
14.2	.0229029	.000524544	32.802
14.3	.0244376	.000597198	32.239
14.4	.0256988	.000660430	31.802
14.5	.0266799	.000711818	31.476
14.6	.0273771	.000749504	31.252
14.7	.0277893	.000772244	31.122
14.8	.0279183	.000779432	31.082
14.9	.0277686	.000771096	31.129
15.0	.0273472	.000747869	31.262
15.1	.0266636	.000710948	31.482
15.2	.0257297	.000662015	31.791
15.3	.0245594	.000603164	32.196
15.4	.0231689	.000536796	32.702
15.5	.0215759	.000465519	33.321
15.6	.0197999	.000392038	34.067
15.7	.0178619	.000319046	34.961
15.8	.0157837	.000249126	36.036
15.9	.0135884	.000184645	37.337
16.0	.0112996	.000127682	38.939
16.1	.00894154	.(4)799512	40.972
16.2	.00653846	.(4)427514	43.690
16.3	.00411473	.(4)169310	47.713
16.4	+.00169447	.(5)287124	55.419
16.5	−.000698693	.(6)488171	63.114
16.6	−.00304182	.(5)925267	50.337
16.7	−.00531286	.(4)282265	45.493
16.8	−.00749086	.(4)561130	42.509
16.9	−.00955612	.(4)913195	40.394
17.0	−.0114904	.000132030	38.793
17.1	−.0132771	.000176280	37.538
17.2	−.0149011	.000222043	36.536
17.3	−.0163495	.000267306	35.730
17.4	−.0176110	.000310149	35.084
17.5	−.0186766	.000348814	34.574
17.6	−.0195389	.000381770	34.182
17.7	−.0201932	.000407764	33.896
17.8	−.0206363	.000425859	33.707
17.9	−.0208676	.000435458	33.610
18.0	−.0208883	.000436322	33.602
18.1	−.0207017	.000428559	33.680
18.2	−.0203130	.000412616	33.845
18.3	−.0197293	.000389245	34.098
18.4	−.0189596	.000359466	34.443
18.5	−.0180144	.000324520	34.888
18.6	−.0169060	.000285811	35.439
18.7	−.0156476	.000244848	36.111
18.8	−.0142542	.000203183	36.921
18.9	−.0127416	.000162348	37.896

TABLE II.—*Continued.*

α	Q_N	Q_N^2	$10 \log_{10} Q_N^{-2}$
19.0	−.0111265	.000123798	39.073
19.1	−.00942635	.(4)888560	40.513
19.2	−.00765927	.(4)586644	42.316
19.3	−.00584365	.(4)341483	44.666
19.4	−.00399811	.(4)159849	47.963
19.5	−.00214124	.(5)458490	53.387
19.6	−.000291487	.(7)849646	70.708
19.7	+.00153306	.(5)235026	56.289
19.8	.00331482	.(4)1098806	49.591
19.9	.00503693	.(4)253706	45.957
20.0	.00668331	.(4)446667	43.500
	Maxima of Q_N^2		
5.135	−.13228	.017498	17.57
8.417	+.064483	.0041580	23.81
11.620	−.040008	.0016006	27.96
14.796	+.027919	.00077945	31.08
17.960	−.020905	.00043703	33.59
21.117	+.016410	.00026928	35.70
24.270	−.013325	.00017756	37.51
27.421	+.011100	.00012320	39.09
30.571	−.0094321	.(4)88964	40.51
33.717	+.0081442	.(4)66327	41.78
36.863	−.0071250	.(4)50766	42.94
40.008	+.0063021	.(4)39717	44.01

TABLE III. Circular case: $m=1$, $n=1$.
$A(u)=1-u^2$, $u<1$.
$A(u)=0$, $u>1$.
$Q_N=G_2=8J_2/\alpha^2$.

α	Q_N	Q_N^2	$10 \log_{10} Q_N^{-2}$
0.0	1.00000	1.00000	0.0000
0.4	.98673	.97364	0.1161
0.8	.94772	.89818	0.4665
1.2	.88527	.78371	1.059
1.6	.80302	.64485	1.905
2.0	.70567	.49797	3.028
2.4	.59858	.35830	4.458
2.8	.48743	.23759	6.242
3.2	.37776	.14270	8.456
3.6	.27457	.075389	11.23
4.0	.18206	.033147	14.80
4.4	.10334	.010679	19.71
4.8	+.040295	.0016237	27.89
5.2	−.0064256	.(4)41288	43.84
5.6	−.037341	.0013943	28.56
6.0	−.053972	.0029130	25.36
6.4	−.058608	.0034349	24.64
6.8	−.054027	.0029189	25.35
7.2	−.043207	.0018668	27.29
7.6	−.029045	.00084360	30.74
8.0	−.014124	.0019949	37.00
8.4	−.00053110	.(6)28207	65.50
8.8	+.010253	.00010513	39.78
9.2	.017392	.00030249	35.19
9.6	.020664	.00042699	33.70
10.0	.020370	.00041495	33.82
10.4	.017212	.00029625	35.28
10.8	.012131	.00014717	38.32
11.2	.0061601	.(4)37946	44.21
11.6	+.00027441	.(7)75301	71.23
12.0	−.0047184	.(4)22263	46.52
12.4	−.0082574	.(4)68185	41.66
12.8	−.010064	.00010129	39.94
13.2	−.010137	.00010276	39.88
13.6	−.0087138	.(4)75929	41.20
14.0	−.0062049	.(4)38501	44.15
14.4	−.0031167	.(5)97139	50.13
14.8	+.(4)30534	.(9)93230	90.30
15.2	.0027753	.(5)77023	51.13
15.6	.0047707	.(4)22760	46.43
16.0	.0058187	.(4)33857	44.70
16.4	.0058795	.(4)34569	44.61
16.8	.0050590	.(4)25594	45.92
17.2	.0035773	.(4)12797	48.93
17.6	+.0017249	.(5)29754	55.26
18.0	−.00018599	.(7)34592	74.61
18.4	−.0018681	.(5)34898	54.57
18.8	−.0030973	.(5)95934	50.18
19.2	−.0037384	.(4)13976	48.55
19.6	−.0037554	.(4)14103	48.51
20.0	−.0032068	.(4)10284	49.88
	Maxima of Q_N^2		
6.379	−.058619	.0034362	24.64
9.760	+.020942	.00043858	33.58
13.017	−.010309	.00010627	39.74
16.224	+.0059706	.(4)35648	44.48
19.410	−.0038235	.(4)14619	48.35
22.583	+.0026225	.(5)68774	51.63

Other maxima at

$\alpha=25.749$, 28.909, 32.050, 35.219 and 38.370, etc.

TABLE IV. Circular case: $m=1$, $n=2$.
$A(u)=(1-u^2)^2$, $u<1$.
$A(u)=0$, $u>1$.
$Q_N=G_3=48J_3/\alpha^3$.

α	Q_N	Q_N^2	$10 \log_{10} Q_N^{-2}$
0.0	1.00000	1.00000	0.0
0.4	.99004	.98018	0.0869
0.8	.96063	.92282	0.3487
1.2	.91318	.83390	0.7889
1.6	.84988	.72230	1.413

TABLE IV.—*Continued.*

α	Q_N	Q_N^2	$10 \log_{10} Q_N^{-2}$
2.0	.77366	.59855	2.229
2.4	.68790	.47320	3.250
2.8	.59628	.35555	4.491
3.2	.50254	.25255	5.977
3.6	.41025	.16830	7.739
4.0	.32263	.10409	9.826
4.4	.24237	.058743	12.31
4.8	.17153	.029423	15.31
5.2	.11146	.012424	19.06
5.6	.062804	.0039443	24.04
6.0	+.025504	.00065046	31.87
6.4	−.0010817	.(5)11701	59.32
6.8	−.018086	.00032710	34.85
7.2	−.026990	.00072844	31.38
7.6	−.029478	.00086894	30.61
8.0	−.027294	.00074494	31.28
8.4	−.022110	.00048886	33.11
8.8	−.015423	.00023786	36.24
9.2	−.0084699	.(4)71739	41.44
9.6	−.0021885	.(5)47896	53.20
10.0	+.0028022	.(5)78524	51.05
10.4	.0061863	.(4)38271	44.17
10.8	.0079133	.(4)62620	42.03
11.2	.0081433	.(4)66314	41.78
11.6	.0071833	.(4)51600	42.87
12.0	.0054205	.(4)29382	45.32
12.4	.0032606	.(4)10631	49.73
12.8	+.0010762	.(5)11583	59.36
13.2	−.00083142	.(6)69126	61.60
13.6	−.0022558	.(5)50888	52.93
14.0	−.0030929	.(5)95659	50.19
14.4	−.0033351	.(4)11123	49.54
14.8	−.0030556	.(5)93369	50.30
15.2	−.0023845	.(5)56856	52.45
15.6	−.0014822	.(5)21968	56.58
16.0	−.00051384	.(6)26403	65.78
16.4	+.00037344	.(6)13946	68.56
16.8	.0010672	.(5)11388	59.44
17.2	.0014991	.(5)22472	56.48
17.6	.0016475	.(5)27143	55.66
18.0	.0015335	.(5)23516	56.29
18.4	.0012116	.(5)14680	58.33
18.8	.00075760	.(6)57396	62.40
19.2	+.00025526	.(7)65160	71.86
19.6	−.00021640	.(7)46830	73.29
20.0	−.00059341	.(6)35213	64.53
	Maxima of Q_N^2		
7.586	−.029480	.00086905	30.61
11.064	+.0082163	.(4)67508	41.71
14.373	−.0033364	.(4)11131	49.53
17.616	+.0016477	.(5)27149	55.66
20.827	−.00092042	.(6)84718	60.72
24.018	+.00056006	.(6)31367	65.04

Other maxima at
$\alpha = 27.20, 30.371, 33.512, 36.699, 39.858$, etc.

TABLE V. Circular case: $m = 1$, $n = 3$.
$A(u) = (1-u^2)^3$, $u < 1$.
$A(u) = 0$, $u > 1$.
$Q_N = G_4 = 384 J_4/\alpha^4$.

α	Q_N	Q_N^2	$10 \log_{10} Q_N^{-2}$
0.0	1.00000	1.00000	0.0
0.4	.99203	.98412	0.0694
0.8	.96842	.93783	0.2788
1.2	.93012	.86513	0.6292
1.6	.87862	.77198	1.124
2.0	.81590	.66569	1.767
2.4	.74429	.55397	2.565
2.8	.66640	.44409	3.525
3.2	.58492	.34213	4.658
3.6	.50251	.25252	5.977
4.0	.42169	.17783	7.500
4.4	.34470	.11882	9.251
4.8	.27341	.074753	11.26
5.2	.20927	.043795	13.59
5.6	.15328	.023496	16.29
6.0	.10597	.011229	19.50
6.4	.067413	.0045446	23.43
6.8	.037309	.0013920	28.56
7.2	+.015016	.00022547	36.47
7.6	−.00035981	.(6)12946	68.88
8.0	−.0098773	.(4)97560	40.11
8.4	−.014680	.00021549	36.67
8.8	−.015915	.00025328	35.96
9.2	−.014667	.00021511	36.67
9.6	−.011902	.00014166	38.49
10.0	−.0084327	.(4)71111	41.48
10.4	−.0048930	.(4)23941	46.21
10.8	−.0017358	.(5)30131	55.21
11.2	+.00075890	.(6)57593	62.40
11.6	.0024645	.(5)60739	52.17
12.0	.0033796	.(4)11422	49.42
12.4	.0035956	.(4)12929	48.88
12.8	.0032638	.(4)10652	49.73
13.2	.0025636	.(5)65718	51.82
13.6	.0016759	.(5)28087	55.51
14.0	+.00076213	.(6)58084	62.36
14.4	−.(4)50560	.(8)25563	85.92
14.8	−.00067630	.(6)45738	63.40
15.2	−.0010720	.(5)11491	59.40
15.6	−.0012333	.(5)15211	58.18
16.0	−.0011874	.(5)14098	58.51
16.4	−.00098264	.(6)96559	60.15
16.8	−.00067889	.(6)46089	63.36
17.2	−.00033720	.(6)11370	69.44
17.6	−.(4)11996	.(9)14389	98.42
18.0	+.00025474	.(7)64892	71.88
18.4	.00043663	.(6)19064	67.20
18.8	.00052353	.(6)27408	65.62
19.2	.00052001	.(6)27041	65.68
19.6	.00044219	.(6)19553	67.09
20.0	.00031361	.(7)98351	70.07

Maxima of Q_N^2 at

$\alpha = 8.780, 12.339, 15.700, 18.982, 22.220, 25.431, 28.628, 31.813, 34.983, 38.160$, etc.

TABLE VI. Circular case: $m=1$, $n=4$.
$A(u)=(1-u^2)^4$, $u<1$.
$A(u)=0$, $u>1$.
$Q_N=G_5=3840J_5/\alpha^5$.

α	Q_N	Q_N^2	$10\log_{10}Q_N^{-2}$
0.0	1.00000	1.00000	0.0
0.4	.99335	.98674	0.0781
0.8	.97364	.94797	0.2321
1.2	.94152	.88646	0.5235
1.6	.89808	.80655	0.9335
2.0	.84476	.71361	1.465
2.4	.78326	.61350	2.122
2.8	.71553	.51198	2.907
3.2	.64359	.41421	3.828
3.6	.56952	.32436	4.890
4.0	.49532	.24535	6.102
4.4	.42285	.17880	7.476
4.8	.35374	.12513	9.026
5.2	.28937	.083736	10.77
5.6	.23081	.053275	12.73
6.0	.17881	.031972	14.95
6.4	.13378	.017898	17.47
6.8	.095840	.0091852	20.37
7.2	.064823	.0042021	23.77
7.6	.040330	.0016265	27.89
8.0	.021770	.00047395	33.24
8.4	+.0084247	.(4)70975	41.49
8.8	−.00050839	.(6)25846	65.88
9.2	−.0058571	.(4)34306	44.65
9.6	−.0084320	.(4)71099	41.48
10.0	−.0089880	.(4)80783	40.93
10.4	−.0081947	.(4)67154	41.73
10.8	−.0066180	.(4)43798	43.59
11.2	−.0047094	.(4)22179	46.54
11.6	−.0028055	.(5)78706	51.04
12.0	−.0011338	.(5)12855	58.91
12.4	+.00017433	.(7)30391	75.17
12.8	.0010681	.(5)11409	59.43
13.2	.0015588	.(5)24297	56.14
13.6	.0017006	.(5)28920	55.39
14.0	.0015735	.(5)24758	56.06
14.4	.0012672	.(5)16058	57.94
14.8	.00086901	.(6)75517	61.22
15.2	.00045446	.(6)20653	66.85
15.6	+.(4)81806	.(8)66923	81.74
16.0	−.00021047	.(7)44299	73.54
16.4	−.00040336	.(6)16270	67.89
16.8	−.00049491	.(6)24494	66.11
17.2	−.00049656	.(6)24657	66.08
17.6	−.00042859	.(6)18369	67.36
18.0	−.00031574	.(7)99695	70.01
18.4	−.00018312	.(7)33533	74.75
18.8	−.(4)52981	.(8)28070	85.52
19.2	+.(4)57453	.(8)33009	84.81
19.6	.00013715	.(7)18810	77.26
20.0	.00018140	.(7)32907	74.83

Maxima of Q_N^2 at

α=9.94, 13.59, 17.00, 20.321, 23.586, 26.820, 30.034, 33.233, 36.422, 39.603, etc.

TABLE VII. Circular case: $m=2$, $n=1$.
$A(u)=1-u^4$, $u<1$.
$A(u)=0$, $u>1$.
$Q_N=\dfrac{3G_2-G_3}{2}$.

α	Q_N	Q_N^2	$10\log_{10}Q_N^{-2}$
0.0	1.00000	1.00000	0.0
0.4	.98508	.97038	0.1305
0.8	.94127	.88598	0.5258
1.2	.87132	.75920	1.197
1.6	.77960	.60777	2.163
2.0	.67167	.45114	3.457
2.4	.55393	.30683	5.131
2.8	.43301	.18750	7.270
3.2	.31536	.099455	10.024
3.6	.20673	.042738	13.69
4.0	.11178	.012495	19.03
4.4	+.033827	.0011443	29.41
4.8	−.025321	.00064115	31.93
5.2	−.065371	.0042733	23.69
5.6	−.087413	.0076410	21.17
6.0	−.093710	.0087815	20.56
6.4	−.087371	.0076337	21.17
6.8	−.071998	.0051837	22.85
7.2	−.051315	.0026333	25.79
7.6	−.028828	.00083107	30.80
8.0	−.0075391	.(4)56838	42.45
8.4	+.010258	.00010523	39.78
8.8	.023091	.00053319	32.73
9.2	.030323	.00091951	30.36
9.6	.032090	.0010298	29.87
10.0	.029155	.00084998	30.71
10.4	.022725	.00051641	32.87
10.8	.014240	.00020279	36.93
11.2	+.0051684	.(4)26713	45.73
11.6	−.0031800	.(4)10113	49.95
12.0	−.0097878	.(4)95800	40.19
12.4	−.014016	.00019646	37.07
12.8	−.015634	.00024443	36.12
13.2	−.014790	.00021874	36.60
13.6	−.011943	.00014263	38.46
14.0	−.0077609	.(4)60232	42.20
14.4	−.0030075	.(5)90451	50.44
14.8	+.0015736	.(5)24763	56.06
15.2	.0053552	.(4)28678	45.42
15.6	.0078972	.(4)62366	42.05
16.0	.0089850	.(4)80730	40.93
16.4	.0086326	.(4)74521	41.28
16.8	.0070550	.(4)49773	43.03
17.2	.0046165	.(4)21312	46.71
17.6	+.0017636	.(5)31104	55.07
18.0	−.0010457	.(5)10936	59.61
18.4	−.0034079	.(4)11614	49.35
18.8	−.0050248	.(4)25248	45.98
19.2	−.0057352	.(4)32893	44.83
19.6	−.0055248	.(4)30524	45.15
20.0	−.0045135	.(4)20372	46.91

TABLE VIII. Circular case: $m=2$, $n=2$.
$A(u)=(1-u^4)^2$, $u<1$;
$A(u)=0$, $u>1$.

$$Q_N=\frac{20G_2-15G_4+3G_6}{8}.$$

α	Q_N	Q_N^2	$10\log_{10}Q_N^{-2}$
0.0	1.00000	1.00000	0.0
0.4	.98756	.97527	0.1089
0.8	.95091	.90424	0.4372
1.2	.89202	.79571	0.9923
1.6	.81407	.66272	1.787
2.0	.72113	.52002	2.840
2.4	.61792	.38183	4.181
2.8	.50952	.25961	5.857
3.2	.40097	.16078	7.938
3.6	.29698	.088200	10.55
4.0	.20164	.040660	13.91
4.4	.11818	.013967	18.55
4.8	+.048840	.0023853	26.22
5.2	−.0052097	.(4)27141	45.67
5.6	−.043840	.0019220	27.16
6.0	−.067876	.0046072	23.37
6.4	−.078937	.0062311	22.05
6.8	−.079230	.0062774	22.02
7.2	−.071320	.0050866	22.94
7.6	−.057896	.0033520	24.75
8.0	−.041550	.0017264	27.63
8.4	−.024592	.00060476	32.18
8.8	−.0089070	.(4)79335	41.01
9.2	+.0041289	.(4)17048	47.68
9.6	.013683	.00018724	37.28
10.0	.019446	.00037816	34.22
10.4	.021567	.00046514	33.32
10.8	.020556	.00042255	33.74
11.2	.017169	.00029478	35.31
11.6	.012285	.00015093	38.21
12.0	.0067895	.(4)46097	43.36
12.4	+.0014750	.(5)21756	56.62
12.8	−.0030284	.(5)91714	50.38
13.2	−.0063007	.(4)39699	44.01
13.6	−.0081442	.(4)66329	41.78
14.0	−.0085711	.(4)73464	41.34
14.4	−.0077678	.(4)60339	42.19
14.8	−.0060452	.(4)36544	44.37
15.2	−.0037808	.(4)14294	48.45
15.6	−.0013623	.(5)18558	57.31
16.0	+.00086276	.(6)74436	61.28
16.4	.0026248	.(5)68896	51.62
16.8	.0037552	.(4)14102	48.51
17.2	.0041937	.(4)17587	47.55
17.6	.0039805	.(4)15845	48.00
18.0	.0032377	.(4)10483	49.80
18.4	.0021416	.(5)45866	53.39
18.8	+.00089251	.(6)79658	60.99
19.2	−.00031531	.(7)99422	70.03
19.6	−.0013187	.(5)17389	57.60
20.0	−.0020035	.(5)40140	53.96

TABLE IX. Circular case: $m=3$, $n=1$.
$A(u)=1-u^6$, $u<1$;
$A(u)=0$, $u>1$.

$$Q_N=\frac{6G_2-4G_3+G_4}{3}.$$

α	Q_N	Q_N^2	$10\log_{10}Q_N^{-2}$
0.0	1.0000	1.0000	0.0
0.4	.9841	.9684	0.139
0.8	.9374	.8787	0.561
1.2	.8630	.7448	1.28
1.6	.7657	.5864	2.32
2.0	.6518	.4248	3.72
2.4	.5281	.2788	5.55
2.8	.4020	.1616	7.92
3.2	.2804	.07865	11.04
3.6	.1697	.02878	15.41
4.0	+.07451	.005552	22.56
4.4	−.001580	.(5)2496	56.03
4.8	−.05698	.003247	24.89
5.2	−.09171	.008410	20.75
5.6	−.1073	.01152	19.39
6.0	−.1066	.01137	19.44
6.4	−.09330	.0087054	20.60
6.8	−.07150	.005113	22.91
7.2	−.04542	.002063	26.85
7.6	−.01890	.0003575	34.47
8.0	+.004852	.(4)2354	46.28
8.4	.02352	.0005534	32.57
8.8	.03577	.001279	28.93
9.2	.04119	.001696	27.71
9.6	.04028	.001622	27.90
10.0	.03419	.001169	29.32
10.4	.02454	.0006024	32.20
10.8	.01313	.0001725	37.63
11.2	+.001715	.(5)2943	55.31
11.6	−.008207	.(4)6736	41.72
12.0	−.01554	.0002414	36.17
12.4	−.01966	.0003867	34.13
12.8	−.02048	.0004192	33.78
13.2	−.01831	.0003353	34.75
13.6	−.01386	.0001921	37.16
14.0	−.008032	.(4)6451	41.90
14.4	−.001803	.(5)3252	54.88
14.8	+.003910	.(4)1529	48.16
15.2	.008373	.(4)7010	41.54
15.6	.01111	.0001234	39.09
16.0	.01193	.0001422	38.47
16.4	.010934	.0001195	39.23
16.8	.008469	.(4)7172	41.44
17.2	.005043	.(4)2544	45.95
17.6	+.001249	.(5)1560	58.07
18.0	−.002332	.(5)5437	52.65
18.4	−.005206	.(4)2710	45.67
18.8	−.007031	.(4)4943	43.06
19.2	−.007644	.(4)5843	42.33
19.6	−.007075	.(4)5005	43.01
20.0	−.005518	.(4)3045	45.16

TABLE X. Circular case: $m=4$, $n=1$.

$A(u)=1-u^2$, $u<1$;
$A(u)=0$, $u>1$.

$$Q_N=\frac{10G_2-10G_3+5G_4-G_5}{4}.$$

α	Q_N	Q_N^2	$10\log_{10}Q_N^{-2}$
0.0	1.0000	1.0000	0.0
0.4	.9834	.9671	0.145
0.8	.9348	.8739	0.585
1.2	.8575	.7353	1.33
1.6	.7566	.5725	2.42
2.0	.6387	.4080	3.89
2.4	.5112	.2614	5.83
2.8	.3820	.1459	8.36
3.2	.2583	.06672	11.76
3.6	.1466	.02149	16.68
4.0	+.05186	.002689	25.70
4.4	−.02241	.0005023	32.99
4.8	−.07476	.005589	22.53
5.2	−.1055	.01112	19.54
5.6	−.1165	.01356	18.68
6.0	−.1109	.01231	19.10
6.4	−.09299	.008648	20.63
6.8	−.06718	.004513	23.46
7.2	−.03798	.001442	28.41
7.6	−.009450	.(4)8930	40.49
8.0	+.01514	.0002291	36.40
8.4	.03349	.001122	29.50
8.8	.04442	.001973	27.05
9.2	.04779	.002283	26.41
9.6	.04436	.001968	27.06
10.0	.03563	.001269	28.97
10.4	.02350	.0005521	32.58
10.8	+.01003	.0001006	39.97
11.2	−.002832	.(5)8020	50.96
11.6	−.01349	.0001820	37.40
12.0	−.02084	.0004343	33.62
12.4	−.02434	.0005926	32.27
12.8	−.02404	.0005778	32.38
13.2	−.02045	.0004182	33.79
13.6	−.01448	.0002095	36.79
14.0	−.007220	.(4)5214	42.83
14.4	+.0001660	.(7)2756	75.60
14.8	.006653	.(4)4426	43.54
15.2	.01145	.0001310	38.83
15.6	.01407	.0001980	37.03
16.0	.01440	.0002074	36.83
16.4	.01264	.0001597	37.97
16.8	.009255	.(4)8565	40.67
17.2	.004898	.(4)2399	46.20
17.6	+.0002857	.(7)8160	70.88
18.0	−.003901	.(4)1522	48.18
18.4	−.007108	.(4)5052	42.97
18.8	−.008970	.(4)8046	40.94
19.2	−.009349	.(4)8739	40.59
19.6	−.008329	.(4)6937	41.59
20.0	−.006187	.(4)3828	44.17

TABLE XI. Linear case: $n=0$.

$A(v)=1$, $v^2\leq 1$.
$A(v)=0$, $v^2>1$.

$$Q_N=H_0=\frac{\sin\beta}{\beta}.$$

β	Q_N	Q_N^2	$10\log_{10}Q_N^{-2}$
0.0	1.000000	1.00000	0.0
0.4	.973546	.94779	0.23287
0.8	.896695	.80406	0.94710
1.2	.776699	.60326	2.1949
1.6	.624734	.39029	4.0861
2.0	.454649	.20671	6.8465
2.4	.281443	.079210	11.012
2.8	+.119639	.014313	18.443
3.2	−.0182419	.00033277	34.779
3.6	−.122922	.015110	18.207
4.0	−.189201	.035797	14.462
4.4	−.216273	.046774	13.300
4.8	−.207534	.043070	13.658
5.2	−.169895	.028864	15.396
5.6	−.112726	.012707	18.960
6.0	−.0465693	.0021687	26.638
6.4	+.0182108	.00033163	34.793
6.8	.0726637	.0052800	22.774
7.2	.110232	.012151	19.154
7.6	.127358	.016220	17.899
8.0	.123670	.015294	18.155
8.4	.101738	.010351	19.850
8.8	.0664679	.0044180	23.548
9.2	+.0242272	.00058696	32.314
9.6	−.0181590	.00032975	34.818
10.0	−.0544021	.0029596	25.288
10.4	−.0795987	.0063360	21.982
10.8	−.0908274	.0082496	20.836
11.2	−.0874266	.0076434	21.167
11.6	−.0709335	.0050316	22.983
12.0	−.0447144	.0019994	26.991
12.4	−.0133552	.00017836	37.487
12.8	+.0180867	.00032713	34.853
13.2	.0448541	.0020119	26.964
13.6	.0631737	.0039909	23.989
14.0	.0707577	.0050066	23.005
14.4	.0670596	.0044970	23.471
14.8	.0532603	.0028367	25.472
15.2	.0319999	.0010240	29.897
15.6	+.00690729	.(4)47711	43.214
16.0	−.0179940	.00032378	34.897
16.4	−.0389089	.0015139	28.199
16.8	−.0528314	.0027912	25.542
17.2	−.0579593	.0033593	24.738
17.6	−.0539116	.0029065	25.366
18.0	−.0417215	.0017407	27.593
18.4	−.0236177	.00055780	32.535
18.8	−.00263487	.(5)69426	51.585
19.2	+.0178810	.00031973	34.952
19.6	.0347941	.0012106	29.170
20.0	.0456473	.0020837	26.812
	Maxima of Q_N^2		
4.49	−.2172	.04719	13.26
7.73	+.1284	.01648	17.83
10.90	−.09132	.008340	20.79
14.07	+.07091	.005029	22.99
17.22	−.05797	.003361	24.74

TABLE XII. Linear case: $m=1$, $n=1$.
$A(v)=1-v^2$, $v^2<1$;
$A(v)=0$, $v^2>1$.
$Q_N=H_1=3(\pi/2\beta)^{\frac{1}{2}}J_{3/2}/\beta$.

β	Q_N	Q_N^2	$10\log_{10}Q_N^{-2}$
0.0	1.000000	1.00000	0.0
0.4	.984091	.96844	0.13930
0.8	.937446	.87881	0.56105
1.2	.863211	.74513	1.2777
1.6	.766328	.58726	2.3117
2.0	.653097	.42654	3.7004
2.4	.530644	.28158	5.5040
2.8	.406324	.16510	7.8225
3.2	.287125	.082441	10.839
3.6	.179129	.032087	14.937
4.0	.0870831	.0075835	21.201
4.4	+.0141105	.00019911	37.009
4.8	−.0384158	.0014758	28.310
5.2	−.0708297	.0050168	22.996
5.6	−.0849769	.0072211	21.414
6.0	−.0838950	.0070384	21.525
6.4	−.0714092	.0050993	22.925
6.8	−.0516912	.0026720	25.732
7.2	−.0288264	.00083096	30.804
7.6	−.00643535	.(4)41414	43.829
8.0	+.0126173	.00015920	37.981
8.4	.0264042	.00069718	31.567
8.8	.0339964	.0011558	29.371
9.2	.0354113	.0012540	29.017
9.6	.0314625	.00098989	30.044
10.0	.0235401	.00055414	32.564
10.4	.0133520	.00017828	37.489
10.8	+.00266210	.(5)70868	51.496
11.2	−.00694591	.(4)48246	43.165
11.6	−.0142514	.00020310	36.923
12.0	−.0185118	.00034269	34.651
12.4	−.0195021	.00038033	34.198
12.8	−.0174819	.00030562	35.148
13.2	−.0131031	.00017169	37.653
13.6	−.00727504	.(4)52926	42.763
14.0	−.00100989	.(5)10199	59.914
14.4	+.00472912	.(4)22365	46.504
14.8	.00915742	.(4)83858	40.765
15.2	.0117608	.00013832	38.591
15.6	.0123408	.00015230	38.173
16.0	.0110117	.00012126	39.163
16.4	.00815406	.(4)66489	41.773
16.8	.00432873	.(4)18738	47.273
17.2	+.000210102	.(7)44143	73.551
17.6	−.00358008	.(4)12817	48.922
18.0	−.00650035	.(4)42255	43.741
18.4	−.00818990	.(4)67074	41.734
18.8	−.00849995	.(4)72249	41.412
19.2	−.00749788	.(4)56218	42.501
19.6	−.00543986	.(4)29592	45.288
20.0	−.00271826	.(5)73889	51.314
	Maxima of Q_N^2		
5.76	−.08617	.007425	21.29
9.10	+.03560	.001268	28.97
12.323	−.01956	.0003826	34.17
15.515	+.01239	.0001534	38.14
18.689	−.008552	.(4)7314	41.36

TABLE XIII. Linear case: $m=1$, $n=2$.
$A(v)=(1-v^2)^2$, $v^2<1$;
$A(v)=0$, $v^2>1$.
$Q_N=H_2=15(\pi/2\beta)^{\frac{1}{2}}J_{5/2}/\beta^2$.

β	Q_N	Q_N^2	$10\log_{10}Q_N^{-2}$
0.0	1.000000	1.00000	0.0
0.4	.988622	.97737	0.09941
0.8	.955091	.91220	0.39910
1.2	.901169	.81211	0.90385
1.6	.829654	.68833	1.622
2.0	.744180	.55380	2.566
2.4	.648961	.42115	3.756
2.8	.548506	.30086	5.216
3.2	.447315	.20009	6.988
3.6	.349596	.12222	9.129
4.0	.259016	.067089	11.73
4.4	.178500	.031862	14.97
4.8	.110103	.012123	19.16
5.2	.0549549	.0030200	25.20
5.6	+.0132729	.00017617	37.54
6.0	−.0155524	.00024188	36.16
6.4	−.0328198	.0010771	29.68
6.8	−.0403401	.0016273	27.89
7.2	−.0402367	.0016190	27.91
7.6	−.0347455	.0012072	29.18
8.0	−.0260279	.00067745	31.69
8.4	−.0160148	.00025647	35.91
8.8	−.00628967	.(4)39560	44.03
9.2	+.00198207	.(5)39286	54.06
9.6	.00807643	.(4)65229	41.86
10.0	.00116913	.00013669	38.64
10.4	.0128907	.00016617	37.79
10.8	.0120228	.00014455	38.40
11.2	.00962381	.(4)92618	40.33
11.6	.00631860	.(4)39925	43.99
12.0	+.00272943	.(5)74498	51.28
12.4	−.000599661	.(6)35959	64.44
12.8	−.00325640	.(4)10604	49.75
13.2	−.00498943	.(4)24894	46.04
13.6	−.00571329	.(4)32642	44.86
14.0	−.00549242	.(4)30167	45.20
14.4	−0.00450886	.(4)20330	46.92
14.8	−.00302019	.(5)91216	50.40
15.2	−.00131400	.(5)17266	57.63
15.6	+.000334905	.(6)11216	69.50
16.0	.00169955	.(5)28885	55.39
16.4	.00262472	.(5)68892	51.62
16.8	.00303785	.(5)92285	50.35
17.2	.00294937	.(5)86988	50.61
17.6	.00243728	.(5)59403	52.26
18.0	.00163061	.(5)26589	55.75
18.4	+.000683533	.(6)46722	63.30
18.8	−.000248914	.(7)61958	72.08
19.2	−.00103267	.(5)10664	59.72
19.6	−.00157098	.(5)24680	56.08
20.0	−.00181371	.(5)32895	54.83
	Maxima of Q_N^2 at		
	$\beta=6.99$, 10.42, 13.70, 16.92, etc.		

TABLE XIV. Linear case: $m=1$, $n=3$.
$A(v)=(1-v^2)^3$, $v^2<1$;
$A(v)=0$, $v^2>1$.
$Q_N=H_3=105(\pi/2\beta)^{\frac{1}{2}}J_{7/2}/\beta^3$.

β	Q_N	Q_N^2	$10\log_{10}Q_N^{-2}$
0.0	1.000000	1.00000	0.0
0.4	.991143	.98237	0.0772
0.8	.964957	.93114	0.3098
1.2	.922570	.85114	0.7000
1.6	.865787	.74959	1.252
2.0	.796978	.63517	1.971
2.4	.718941	.51688	2.866
2.8	.634739	.40289	3.948
3.2	.547524	.29978	5.232
3.6	.460367	.21194	6.738
4.0	.376103	.14145	8.494
4.4	.297191	.088323	10.54
4.8	.225615	.050902	12.93
5.2	.162813	.026508	15.77
5.6	.109654	.012024	19.20
6.0	.0664442	.0044148	23.55
6.4	.0329743	.0010873	29.64
6.8	+.00859192	.(4)73821	41.32
7.2	−.00770373	.(4)59347	42.27
7.6	−.0171547	.00029428	35.31
8.0	−.0211341	.00044665	33.50
8.4	−.0210412	.00044273	33.54
8.8	−.0182078	.00033152	34.79
9.2	−.0138235	.00019109	37.19
9.6	−.00888144	.(4)78880	41.03
10.0	−.00414706	.(4)17198	47.65
10.4	−.000149278	.(7)22284	76.52
10.8	+.00280886	.(5)78897	51.03
11.2	.00462325	.(4)21374	46.70
11.6	.00535040	.(4)28627	45.43
12.0	.00516281	.(4)26655	45.74
12.4	.00430271	.(4)18513	47.33
12.8	.00303890	.(5)92349	50.35
13.2	.00162982	.(5)26563	55.76
13.6	+.000295529	.(7)87338	70.59
14.0	−.000800451	.(6)64072	61.93
14.4	−.00155927	.(5)24313	56.14
14.8	−.00194584	.(5)37863	54.22
15.2	−.00198068	.(5)39231	54.06
15.6	−.00172668	.(5)29814	55.26
16.0	−.00127315	.(5)16209	57.90
16.4	−.000719538	.(6)51774	62.86
16.8	−.000160080	.(7)25626	75.91
17.2	+.000324075	.(6)10502	69.79
17.6	.000679906	.(6)46227	63.35
18.0	.000878345	.(6)77149	61.13
18.4	.000917327	.(6)84149	60.75
18.8	.000817073	.(6)66761	61.75
19.2	.000613830	.(6)37679	64.24
19.6	.000352485	.(6)12425	69.06
20.0	.(4)791485	.(8)62645	82.03

Maxima of Q_N^2 at
$\beta=8.18$, 11.71, 15.04, 18.30, etc.

TABLE XV. Linear case: $m=1$, $n=4$.
$A(v)=(1-v^2)^4$, $v^2<1$;
$A(v)=0$, $v^2>1$.
$Q_N=H_4=945(\pi/2\beta)^{\frac{1}{2}}J_{9/2}/\beta^4$.

β	Q_N	Q_N^2	$10\log_{10}Q_N^{-2}$
0.0	1.000000	1.00000	0.0
0.4	.992725	.98550	0.0634
0.8	.971265	.94336	0.2531
1.2	.936329	.87671	0.5714
1.6	.889205	.79069	1.020
2.0	.831564	.69150	1.602
2.4	.765403	.58584	2.322
2.8	.692944	.48017	3.186
3.2	.616520	.38010	4.201
3.6	.538471	.28995	5.377
4.0	.461031	.21255	6.725
4.4	.386238	.14918	8.263
4.8	.315852	.099762	10.01
5.2	.251296	.063150	12.00
5.6	.193622	.037489	14.26
6.0	.143494	.020591	16.86
6.4	.1011971	.0102409	19.90
6.8	.0666677	.0044446	23.52
7.2	.0395366	.0015631	28.06
7.6	.0191866	.00036813	34.34
8.0	+.00481733	.(4)23207	46.34
8.4	−.00448781	.(4)20140	46.96
8.8	−.00969581	.(4)94009	40.27
9.2	−.0117646	.00013841	38.59
9.6	−.0115923	.00013438	38.72
10.0	−.00997819	.(4)99564	40.02
10.4	−.00759542	.(4)57690	42.39
10.8	−.00497668	.(4)24767	46.06
11.2	−.00251144	.(5)63073	52.00
11.6	−.000453304	.(6)20548	66.87
12.0	+.001064602	.(5)11334	59.46
12.4	.00200865	.(5)40347	53.94
12.8	.00242068	.(5)58597	52.32
13.2	.00239332	.(5)57280	52.42
13.6	.00204669	.(5)41889	53.78
14.0	.00150813	.(5)22745	56.43
14.4	.000896143	.(6)80307	60.95
14.8	+.000309004	.(7)95483	70.20
15.2	−.000181791	.(7)33048	74.81
15.6	−.000533696	.(6)28483	65.45
16.0	−.000731562	.(6)53518	62.71
16.4	−.000783345	.(6)61363	62.12
16.8	−.000713823	.(6)50954	62.93
17.2	−.000559064	.(6)31255	65.05
17.6	−.000357421	.(6)12775	68.94
18.0	−.000146274	.(7)21396	76.70
18.4	+.(4)435049	.(8)18927	87.23
18.8	.000190010	.(7)36104	74.42
19.2	.000281384	.(7)79177	71.01
19.6	.000315438	.(7)99501	70.02
20.0	.000298125	.(7)88879	70.51

TABLE XVI. Linear case: $m=2$, $n=1$.
$A(v)=1-v^4$, $v^2<1$;
$A(v)=0$, $v^2>1$.

$$Q_N=\frac{5H_1-2H_2}{3}.$$

β	Q_N	Q_N^2	$10\log_{10}Q_N^{-2}$
0.0	1.00000	1.00000	0.0
0.4	.98107	.96250	0.1660
0.8	.92568	.85689	0.6707
1.2	.83791	.70209	1.536
1.6	.72411	.52434	2.804
2.0	.59238	.35091	4.548
2.4	.45177	.20409	6.902
2.8	.31154	.097055	10.130
3.2	.18033	.032519	14.88
3.6	+.065484	.0042882	23.68
4.0	−.027539	.00075839	31.20
4.4	−.095483	.0091169	20.40
4.8	−.13743	.018886	17.24
5.2	−.15469	.023928	16.21
5.6	−.15048	.022643	16.45
6.0	−.12946	.016759	17.76
6.4	−.097135	.0094353	20.25
6.8	−.059259	.0035116	24.54
7.2	−.021220	.00045027	33.47
7.6	+.012438	.00015471	38.11
8.0	.038381	.0014731	28.32
8.4	.054684	.0029903	25.24
8.8	.060854	.0037032	24.31
9.2	.057697	.0033290	24.78
9.6	.047053	.0022140	26.55
10.0	.031439	.00098841	30.05
10.4	+.013660	.00018659	37.29
10.8	−.0035784	.(4)12805	48.93
11.2	−.017992	.00032373	34.90
11.6	−.027965	.00078202	31.07
12.0	−.032673	.00106750	29.72
12.4	−.032104	.00103065	29.87
12.8	−.026966	.00072714	31.38
13.2	−.018512	.00034270	34.65
13.6	−.0083162	.(4)69159	41.60
14.0	+.0019785	.(5)39143	54.07
14.4	.010888	.00011854	39.26
14.8	.017276	.00029845	35.25
15.2	.020477	.000419320	33.77
15.6	.020345	.00041391	33.83
16.0	.017220	.00029652	35.28
16.4	.011840	.00014019	38.53
16.8	+.0051893	.(4)26929	45.70
17.2	−.0016161	.(5)26117	55.83
17.6	−.0075917	.(4)57633	42.39
18.0	−.011921	.00014211	38.47
18.4	−.014106	.00019897	37.01
18.8	−.014001	.00019602	37.08
19.2	−.011808	.00013943	38.56
19.6	−.0080191	.(4)64306	41.92
20.0	−.0033213	.(4)11031	49.57

TABLE XVII. Linear case: $m=2$, $n=2$.
$A(v)=(1-v^4)^2$, $v^2<1$;
$A(v)=0$, $v^2>1$.

$$Q_N=\frac{21H_2-18H_3+4H_4}{7}.$$

β	Q_N	Q_N^2	$10\log_{10}Q_N^{-2}$
0.0	1.00000	1.00000	0.0
0.4	.98449	.96922	0.1358
0.8	.93896	.88165	0.5469
1.2	.86623	.75035	1.247
1.6	.77077	.59409	2.261
2.0	.65835	.43342	3.631
2.4	.53555	.28681	5.424
2.8	.40930	.16753	7.759
3.2	.28632	.081981	10.863
3.6	.17268	.029820	15.25
4.0	+.073372	.0053835	22.69
4.4	−.0079980	.(4)63968	41.94
4.8	−.069357	.0048104	23.18
5.2	−.11020	.012144	19.16
5.6	−.13151	.017294	17.62
6.0	−.13552	.018365	17.36
6.4	−.12542	.015730	18.03
6.8	−.10502	.011029	19.57
7.2	−.078308	.0061322	22.12
7.6	−.049161	.0024168	26.17
8.0	−.020986	.00044042	33.56
8.4	+.0034971	.(4)12230	49.13
8.8	.022411	.00050223	32.99
9.2	.034770	.0012089	29.18
9.6	.040443	.0016356	27.86
10.0	.040036	.0016029	27.95
10.4	.034716	.0012052	29.19
10.8	.026002	.00067609	31.70
11.2	.015548	.00024174	36.17
11.6	+.0049386	.(4)24390	46.13
12.0	−.0044792	.(4)20063	46.98
12.4	−.011715	.00013725	38.62
12.8	−.016200	.00026245	35.81
13.2	−.017792	.00031654	35.00
13.6	−.016730	.00027989	35.53
14.0	−.013557	.00018380	37.36
14.4	−.0090049	.(4)81089	40.91
14.8	−.0038805	.(4)15058	48.22
15.2	+.0010473	.(5)10968	59.60
15.6	.0051398	.(4)26417	45.78
16.0	.0079544	.(4)63273	41.99
16.4	.0092768	.(4)86058	40.65
16.8	.0091173	.(4)83125	40.80
17.2	.0076953	.(4)59218	42.28
17.6	.0053593	.(4)28722	45.42
18.0	+.0025496	.(5)65007	51.87
18.4	−.00028338	.(7)80305	70.95
18.8	−.0027392	.(5)75033	51.25
19.2	−.0045156	.(4)20391	46.91
19.6	−.0054391	.(4)29584	45.29
20.0	−.0054743	.(4)29968	45.23

TABLE XVIII. Linear case: $m=3$, $n=1$.
$A(v)=1-v^6$, $v^2<1$;
$A(v)=0$, $v^2>1$.

$$Q_N=\frac{35H_1-28H_2+8H_3}{15}.$$

β	Q_N	Q_N^2	$10\log_{10}Q_N^{-2}$
0.0	1.00000	1.00000	0.0
0.4	.97939	.95921	0.1808
0.8	.91918	.84489	0.7321
1.2	.82401	.67900	1.681
1.6	.70116	.49163	3.084
2.0	.55981	.31339	5.039
2.4	.41021	.16827	7.740
2.8	.26274	.069032	11.61
3.2	.12698	.016125	17.93
3.6	+.0109175	.00011919	39.24
4.0	−.079714	.0063544	21.97
4.4	−.14177	.020100	16.97
4.8	−.17483	.030567	15.15
5.2	−.18102	.032768	14.85
5.6	−.16457	.027084	15.67
6.0	−.13129	.017236	17.64
6.4	−.087772	.0077038	21.13
6.8	−.040729	.0016588	27.80
7.2	+.0037383	.(4)13975	48.55
7.6	.040693	.0016559	27.81
8.0	.066754	.0044561	23.51
8.4	.080282	.0064452	21.91
8.8	.081355	.0066186	21.79
9.2	.071554	.0051200	22.91
9.6	.053600	.0028729	25.42
10.0	.030891	.00095428	30.20
10.4	+.0070124	.(4)49174	43.08
10.8	−.014733	.00021706	36.63
11.2	−.031706	.00100526	29.98
11.6	−.042194	.0017804	27.49
12.0	−.045536	.0020735	26.83
12.4	−.042091	.0017716	27.52
12.8	−.033092	.00109506	29.61
13.2	−.020391	.00041580	33.81
13.6	−.0061527	.(4)37855	44.22
14.0	+.0074692	.(4)55789	42.53
14.4	.018620	.00034669	34.60
14.8	.025967	.00067430	31.71
15.2	.028838	.00083165	30.80
15.6	.027249	.00074251	31.29
16.0	.021842	.00047709	33.21
16.4	.013743	.00018887	37.24
16.8	+.0043443	.(4)18873	47.24
17.2	−.0048424	.(4)23449	46.30
17.6	−.012540	.00015726	38.03
18.0	−.017743	.00031481	35.02
18.4	−.019896	.00039587	34.02
18.8	−.019862	.00039450	34.04
19.2	−.015240	.00023226	36.34
19.6	−.0095725	.(4)91633	40.38
20.0	−.0029148	.(5)84961	50.71

TABLE XIX. Linear case: $m=4$, $n=1$.
$A(v)=1-v^8$, $v^2<1$;
$A(v)=0$, $v^2>1$.

$$Q_N=\frac{105H_1-126H_2+72H_3-16H_4}{35}.$$

β	Q_N	Q_N^2	$10\log_{10}Q_N^{-2}$
0.0	1.00000	1.00000	0.0
0.4	.97833	.95714	0.1902
0.8	.91506	.83733	0.7711
1.2	.81525	.66463	1.774
1.6	.68678	.47167	3.264
2.0	.53960	.29116	5.359
2.4	.38474	.14802	8.297
2.8	.23332	.054441	12.64
3.2	+.095538	.0091276	20.40
3.6	−.020276	.00041112	33.86
4.0	−.10827	.011722	19.31
4.4	−.16547	.027380	15.63
4.8	−.19189	.036820	14.34
5.2	−.19028	.036205	14.41
5.6	−.16565	.027441	15.62
6.0	−.12461	.015527	18.09
6.4	−.074505	.0055510	22.56
6.8	−.022651	.00051307	32.90
7.2	+.024451	.00059787	32.23
7.6	.061717	.0038090	24.19
8.0	.085874	.0073744	21.32
8.4	.095633	.0091456	20.39
8.8	.091608	.0083921	20.76
9.2	.076040	.0057820	22.38
9.6	.052341	.0027396	25.62
10.0	+.024562	.00060329	32.19
10.4	−.0031854	.(4)10147	49.94
10.8	−.027243	.00074215	31.29
11.2	−.044825	.0020093	26.97
11.6	−.054287	.0029471	25.31
12.0	−.055227	.0030501	25.16
12.4	−.048415	.0023440	26.30
12.8	−.035578	.0012658	28.98
13.2	−.019089	.00036439	34.38
13.6	−.0015850	.(5)25121	56.00
14.0	+.014407	.00020756	36.83
14.4	.026802	.00071834	31.44
14.8	.034201	.0011697	29.32
15.2	.036021	.0012975	28.87
15.6	.032509	.0010568	29.76
16.0	.024632	.00060674	32.17
16.4	.013891	.00019296	37.15
16.8	+.0020469	.(5)41902	53.78
17.2	−.0090652	.(4)82178	40.85
17.6	−.017952	.00032229	34.92
18.0	−.023497	.00055213	32.58
18.4	−.025163	.00063319	31.98
18.8	−.023010	.00052945	32.76
19.2	−.017642	.00031124	35.07
19.6	−.010083	.00010167	39.93
20.0	−.0015989	.(5)25564	55.92

TABLE XX. Positions of the first null, magnitudes of the first minor lobe, and the efficiencies of the patterns tabulated in Tables I to XIX.

		Circular case			Linear case		
m	n	α_0	db_1	E	β_0	db_1	E
1	−1	2.4	7.9	—	1.6	0.0	—
—	0	3.8	17.6	1	3.1	13.3	1
1	1	5.1	24.6	1/2	4.5	21.3	2/2
1	2	6.4	30.6	1/3	5.8	27.7	8/15
1	3	7.6	36.0	1/4	7.0	33.4	16/35
1	4	8.8	40.9	1/5	8.2	38.5	128/315
2	1	4.6	20.6	2/3	3.9	16.1	4/5
2	2	5.2	22.0	8/15	4.4	17.3	32/45
3	1	4.4	19.4	3/4	3.6+	14.8	6/7
4	1	4.3	18.7	4/5	3.6−	14.3	8/9

TABLE XXI. Linear case: $A(v)=\cos(\pi v/2)$, $v^2<1$. $A(v)=0$, $v^2>1$.

$$Q_N=\frac{\cos\beta}{1-(2\beta/\pi)^2}.$$

β	Q_N	Q_N^2	$10\log_{10}Q_N^{-2}$
0.0	1.00000	1.00000	0.0000
0.4	.98493	.97009	0.1319
0.8	.94071	.88494	0.5308
1.2	.87024	.75731	1.207
1.6	.77805	.60537	2.180
2.0	.66997	.44887	3.479
2.4	.55259	.30535	5.152
2.8	.43272	.18725	7.276
3.2	.31691	.100430	9.981
3.6	.21088	.044470	13.52
4.0	.11918	.014204	18.48
4.4	+.044890	.0020151	26.96
4.8	−.010494	.00011013	39.58
5.2	−.047045	.0022132	26.55
5.6	−.066233	.0043868	23.58
6.0	−.070651	.0049916	23.02
6.4	−.063664	.0040531	23.92
6.8	−.049007	.0024017	26.19
7.2	−.030402	.00092431	30.34
7.6	−.011212	.00012571	39.01
8.0	+.0058344	.(4)34040	44.68
8.4	.018817	.00035408	34.51
8.8	.026694	.00071255	31.47
9.2	.029272	.00085683	30.67
9.6	.027088	.00073378	31.34
10.0	.021227	.00045059	33.46
10.4	.013096	.00017151	37.66
10.8	+.0041997	.(4)17637	47.54
11.2	−.0040732	.(4)16591	47.80
11.6	−.0106153	.00011268	39.48
12.0	−.014711	.00021642	36.65
12.4	−.016084	.00025868	35.87
12.8	−.014875	.00022126	36.55
13.2	−.011576	.00013400	38.73
13.6	−.0069185	.(4)47866	43.20
14.0	−.0017433	.(5)30391	55.17
14.4	+.0031288	.(5)97896	50.09
14.8	.0070107	.(4)49150	43.08
15.2	.0094318	.(4)88960	40.51
15.6	.0101831	.00010370	39.84
16.0	.0093200	.(4)86863	40.61
16.4	.0071288	.(4)50820	42.94
16.8	.0040629	.(4)16507	47.82
17.2	+.00066172	.(6)43787	63.59
17.6	−.0025353	.(5)64275	51.92
18.0	−.0050672	.(4)25676	45.90
18.4	−.0066120	.(4)43718	43.59
18.8	−.0070216	.(4)49302	43.07
19.2	−.0063288	.(4)40054	43.97
19.6	−.0047279	.(4)22354	46.51
20.0	−.0025329	.(5)64155	51.93

TABLE XXII. Linear case: $A(v)=\cos^2(\pi v/2)$, $v^2<1$. $A(v)=0$, $v^2>1$.

$$Q_N=\frac{\sin\beta}{\beta(1-(\beta/\pi)^2)}.$$

β	Q_N	Q_N^2	$10\log_{10}Q_N^{-2}$
0.0	1.00000	1.00000	0.0
0.4	.98959	.97928	0.09094
0.8	.95887	.91944	0.3648
1.2	.90938	.82697	0.8250
1.6	.84353	.71154	1.478
2.0	.76448	.58443	2.333
2.4	.67591	.45686	3.402
2.8	.58178	.33847	4.705
3.2	.48608	.23627	6.266
3.6	.39257	.15411	8.122
4.0	.30460	.092783	10.325
4.4	.22491	.050587	12.96
4.8	.15552	.024187	16.14
5.2	.097656	.0095368	20.21
5.6	.051770	.0026802	25.72
6.0	+.017589	.00030939	35.10
6.4	−.0057810	.(4)33420	44.76
6.8	−.019718	.00038881	34.10
7.2	−.025922	.00067200	31.73
7.6	−.026247	.00068889	31.62
8.0	−.022549	.00050844	32.94
8.4	−.016545	.00027373	35.63
8.8	−.0097086	.(4)94256	40.26
9.2	−.0031980	.(4)10227	49.90
9.6	+.0021779	.(5)47434	53.24
10.0	.0059572	.(4)35489	44.50
10.4	.0079927	.(4)63884	41.95
10.8	.0083959	.(4)70491	41.52
11.2	.0074661	.(4)55743	42.54
11.6	.0056146	.(4)31524	45.01
12.0	.0032902	.(4)10825	49.66
12.4	+.00091605	.(6)83914	60.76
12.8	−.0011594	.(5)13441	58.72
13.2	−.0026933	.(5)72536	51.39
13.6	−.0035610	.(4)12681	48.97
14.0	−.0037519	.(4)14077	48.51
14.4	−.0033513	.(4)11231	49.50
14.8	−.0025131	.(5)63155	52.00
15.2	−.0014280	.(5)20391	56.91
15.6	−.00029197	.(7)85246	70.69

TABLE XXII.—*Continued.*

β	Q_N	Q_N^2	$10 \log_{10} Q_N^{-2}$
16.0	+.00072154	.(6)52062	62.84
16.4	.0014822	.(5)21968	56.58
16.8	.0019144	.(5)36649	54.36
17.2	.0020003	.(5)40013	53.98
17.6	.0017743	.(5)31480	55.02
18.0	.0013108	.(5)17183	57.65
18.4	.00070917	.(6)50292	62.99
18.8	+.(4)75691	.(8)57291	82.42
19.2	−.00049190	.(6)24196	66.16
19.6	−.00091748	.(6)84177	60.75
20.0	−.0011548	.(5)13335	58.75

Basically, the tables are tabulations of the function Q_N, of its square, and of ten times the logarithm (to the base ten) of its inverse square. The function Q_N is different for each table, and is stated explicitly at the beginning of each table.

The function Q_N may be regarded as the directional pattern at infinity of the continuous source distribution which is stated for each table. In Tables I to X, the function $A(u)$ is the relative source density on the surface of a circle of radius a. The argument u is equal to r/a. The function $A(u)$ is so normalized that it has the value unity at the center of the circle. The argument α is equal to $ka \sin \theta$, where k is equal to 2π divided by the wave-length, and where θ is the angle between the direction and the line which is normal to the surface.

In Tables XI to XXII, exclusive of Table XX, the function $A(v)$ is the relative source density along a line of length $2b$. The argument v is equal to x/b, where x is measured from the center of the line. The function $A(v)$ is so normalized that it has the value unity at the center of the line. The argument β is equal to $kb \sin \theta$, where θ is now the angle between the direction and the *plane* which is normal to the line.

It is evident that if there are no other sources to be considered, then at large distances from the source Q_N^2 is proportional to the intensity along a radial line. Under these conditions, the last column indicates the number of decibels by which the intensity in a given direction is below the intensity in the direction $\theta=0$ at the same (large) distance from the source.

In Table XX is tabulated the efficiency of each of the source distributions in the preceding 19 tables, as well as the possible of the first null of the pattern, and the number of decibels by which the maximum of the first minor lobe is less than the maximum of the principal lobe. The information given in Table XX is also contained in Figs. 3 to 6. The efficiencies of the distributions described in Tables XXI and XXII are 0.6366 and 0.5, respectively.

The Dipole Microphone

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A DIPOLE microphone is a microphone in which the response is a function of the sound pressure at two distinct points. In the particular model to be discussed in this paper, the sound pressure at each of the two selected points is transmitted through tubes to opposite sides of the microphone diaphragm. The use of tubes makes it possible to remove the microphone transducer element from a location directly in front of the talker's mouth, and yet retain the acoustical advantage of a close-talking microphone. Figure 1 shows the dipole microphone as a part of a telephone operator's set, and Fig. 2 shows an exploded view of the same set.

FIG. 1. The dipole microphone set.

The capsule containing the microphone diaphragm and the electromagnetic transducer is mounted in the same case with the telephone receiver. By careful construction it has been possible to eliminate any trouble caused by coupling between the receiver and microphone. The transparent plastic mouthpiece at the free ends of the sound conducting tubes is called the

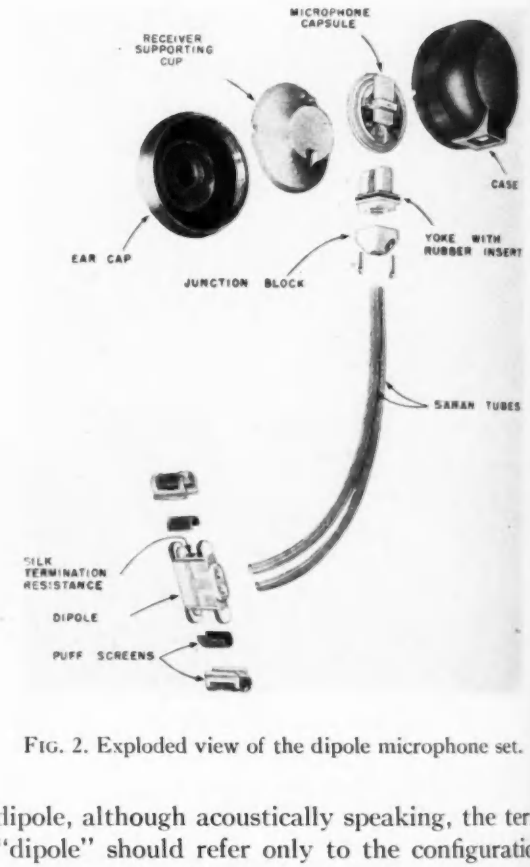

FIG. 2. Exploded view of the dipole microphone set.

dipole, although acoustically speaking, the term "dipole" should refer only to the configuration constituted by the two tube openings at the top and bottom of the plastic mouthpiece. A disk of silk cloth, which forms an acoustic resistance, covers each of the two dipole openings. The silk disks eliminate the peaks in the response curve caused by standing waves in the tubes. Two puff screens of fine mesh wire screen cover each end of the dipole. When the dipole is used very close to the mouth, these puff screens are necessary to reduce the blasting noises accompanying such explosive sounds as "p" and "t."

ANALYSIS

To analyze the performance of such a microphone, let us consider the analogous electric circuit shown in Fig. 3 where: p_1 = sound pressure

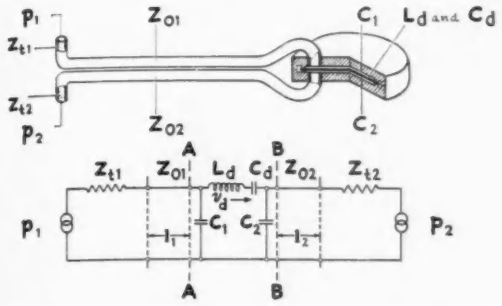

FIG. 3. Schematic diagram of the acoustical system and an analogous electrical circuit.

at one end of the dipole, p_2 = sound pressure at the other end of the dipole, Z_{t1} = acoustic impedance terminating the first tube, including the radiation impedance, Z_{t2} = acoustic impedance terminating the second tube, including the radiation impedance, Z_{01} = characteristic acoustic impedance of the first tube, Z_{02} = characteristic acoustic impedance of the second tube, $\gamma_1 = \alpha_1 + j\beta_1$ = propagation constant of the first tube, $\gamma_2 = \alpha_2 + j\beta_2$ = propagation constant of the second tube, l_1 = length of the first tube, l_2 = length of the second tube, C_1 = acoustic compliance of the cavity on one side of the diaphragm, C_2 = acoustic compliance of the cavity on the other side of the diaphragm, L_d = acoustic inertance of the diaphragm, C_d = acoustic compliance of the diaphragm, v_d = volume velocity of the diaphragm. As this circuit can be treated by fairly straightforward analysis, the method will merely be sketched and the results given. If we enter the circuit at the line A–A we can, by using Thevenin's theorem, replace all of the circuit elements to the left of A–A by an equivalent generator having a certain open circuit voltage and a certain internal impedance. This internal impedance can be expressed by:

$$Z_{AA} = Z_{01} \tanh (\gamma_1 l_1 + \gamma_{t1}), \tag{1}$$

where

$$\gamma_{t1} = \tanh^{-1} \frac{Z_{t1}}{Z_{01}}.$$

If $Z_{t1} = Z_{01}$, $\gamma_{t1} = \infty$, and Eq. (1) reduces to

$$Z_{AA} = Z_{01}. \tag{2}$$

In other words, the internal impedance of the equivalent generator is that of a transmission line terminated in its own characteristic impedance, and is, of course, substantially resistive and independent of frequency. The open circuit voltage at A–A, or, if we refer to the acoustical diagram, the blocked tube pressure at the cavity end of the top tube, is expressed by

$$p_{AA} = p_1 \frac{\cosh \gamma_{t1}}{\cosh (\gamma_{t1} + \gamma_1 l_1)}. \tag{3}$$

Now, if we still assume that $Z_{t1} = Z_{01}$ we can rewrite Eq. (3) in the exponential form,

$$p_{AA} = p_1 \exp (-\gamma_1 l_1)$$
$$= p_1 \exp (-\alpha_1 l_1) \exp (-j\beta_1 l_1). \tag{4}$$

From Eq. (4) it is evident that, if the attenuation in the tube is small, as it is in the practical microphone, the major effect of the tube is the introduction of a phase shift and not a change in amplitude. Since the equivalent generator has a resistive internal impedance and an open circuit voltage which are both substantially independent of frequency, there are no resonance peaks introduced by the tube.

The right side of the microphone may be treated similarly, and another equivalent generator substituted for all of the circuit elements to the right of the line B–B. If the microphone be assumed perfectly symmetrical, if attenuation be neglected, and if the termination impedances match the characteristic impedances of the tubes, a much simpler circuit can be drawn as shown in Fig. 4, where $\Delta p' = (p_1 - p_2)\epsilon^{-j\beta l}$. That is, as far

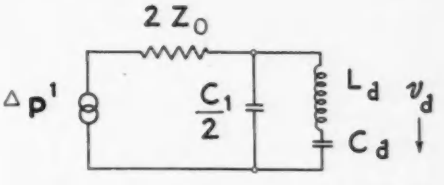

FIG. 4. Simplified equivalent circuit.

as amplitude is concerned, $\Delta p'$ is equal to the

complex difference in pressure between the two ends of the dipole.

Since not all of the assumptions made in the simplified circuit are true for the practical microphone, the simplified circuit cannot be used for rigorous analysis. It can be used, however, to show the effect of the tubes on the sensitivity of the microphone. Since the tubes are fairly small, a little over $\frac{1}{8}''$ I.D., one might suppose the use of larger tubes would increase the sensitivity. If $2Z_0$ is small compared to the parallel impedance of $C_1/2$, L_d and C_d, the velocity through L_d and C_d becomes substantially independent of both Z_0 and C_1. In the practical microphone $2Z_0$ is small compared to the parallel circuit impedance over most of the voice frequency range. Hence, there would be no particular gain in sensitivity if the tubes were larger. At diaphragm resonance, however, the impedance in the L_d, C_d branch is very low and the diaphragm velocity is controlled by the value of Z_0. Thus by the proper choice of circuit constants it is possible to control the relative height of the diaphragm resonance peak.

FREQUENCY RESPONSE

To calculate the frequency response of the microphone, it was necessary to use the first analogous circuit (Fig. 3) since in the model considered, the cavities on either side of the diaphragm did not have the same volume. The calculations for the dashed curve in Fig. 5 were

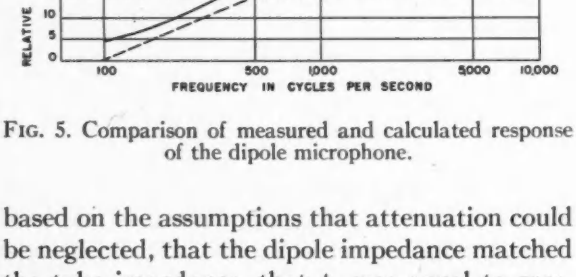

FIG. 5. Comparison of measured and calculated response of the dipole microphone.

based on the assumptions that attenuation could be neglected, that the dipole impedance matched the tube impedance, that p_2 was equal to zero, and that p_1 was constant and independent of frequency. Later in this paper, we shall investigate the pressure at the ends of the dipole more thoroughly. Since the microphone transducer was a magnetic diaphragm type unit, its open-circuit voltage was assumed to be proportional to the diaphragm velocity. The individual points were calculated after taking into account the effect of attenuation[1] on the characteristic impedance of the tube and on the propagation constant. We also used the measured value of the dipole impedance instead of assuming that it matched the tube impedance exactly. The solid curve represents the measured frequency response. The measured curve shows greater high frequency response than the calculations predicted. Unfortunately, there has been no opportunity to investigate the reasons for this discrepancy. Notice, however, that a curve following the calculated points departs from a straight line at about the same frequencies as the measured curve, and the variations are of the same order of magnitude.

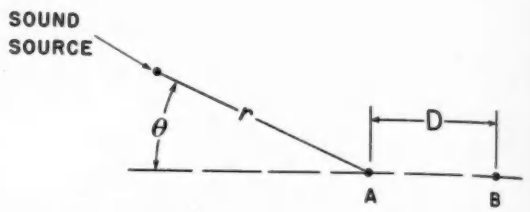

FIG. 6. Schematic representation of a sound source and a dipole microphone.

NOISE REDUCTION

Now let us examine the pressure at each end of the dipole a little more closely. Figure 6 shows a source of sound which for simplicity we can assume to be a point source. Of course, such an assumption means we are neglecting diffraction about the wearer's head, but it will serve as a start. At a distance r from the source is located one end of the dipole A. The other end of the dipole is located at B. Since the symmetrical microphone responds to the difference in pressure between the two ends of the dipole, let us work out an expression for the difference in pressure between A and B.

[1] W. P. Mason, *Electromechanical Transducers and Wave Filters* (D. Van Nostrand Company, Inc., New York, 1942), p. 118.

We can express the pressure at A by

$$p_A = \frac{p}{r} \sin k(ct-r), \tag{5}$$

where p = sound pressure produced by the source at a unit distance, $k = \omega/c$, $\omega = 2\pi f$, c = velocity of sound. If θ is small or r is large compared to D, the pressure at B can be expressed by

$$p_B = \frac{p}{r + D\cos\theta} \sin k(ct-r-D\cos\theta). \tag{6}$$

To obtain the difference in pressure, it is necessary merely to subtract Eq. (6) from Eq. (5). Then

$$\Delta p = \frac{p}{r} \sin k(ct-r) - \frac{p}{r+D\cos\theta} \times \sin k(ct-r-D\cos\theta). \tag{7}$$

Two extreme conditions are of interest in practice. One applies when the sound source is very close to one end of the dipole and the other, when the source is a long distance away. For close-up use, r is small compared to D and Eq. (7) becomes

$$\Delta p \cong p/r \sin k(ct-r). \tag{8}$$

That is, the pressure difference between the ends of the dipole is substantially equal to the pressure at A, and the pressure at B can be neglected. The measured frequency response shown in Fig. 5 was obtained by placing one end of the dipole so close to the sound source that Eq. (8) was applicable. For distant sounds, r is large compared to D and Eq. (7) becomes

$$\Delta p \cong 2 \sin \frac{kD\cos\theta}{2} \frac{p}{r} \cos k\left(ct - r - \frac{D}{2}\cos\theta\right). \tag{9}$$

If we compare Eqs. (6) and (9) we can see that, as far as amplitude is concerned, the difference in pressure for distant sounds is equal to the pressure at one end of the dipole multiplied by the term $2 \sin [(kD\cos\theta)/2]$. Since this term becomes progressively smaller for low frequencies, it is apparent that the dipole microphone discriminates against low frequency sounds arriving from a distance. Also, since if $\theta = \pi/2$, $\cos\theta = 0$, the microphone does not respond to sounds arriving from a direction perpendicular to the line connecting the two ends of the dipole. In the operator's set, the dipole is so oriented that the voices of the adjacent operators, and also the sound of the clattering plugs, arrive approximately from the direction of minium response. For frequencies low enough that $\sin kD \cong kD$, the dipole microphone shows the familiar cosine directional characteristics which are inherent in any pressure gradient microphone. The measured directional characteristics of an early experimental dipole microphone are shown in Fig. 7.

Two effects, then, the directional characteristics and the loss of low frequency sensitivity with distance, tend to discriminate against sounds which do not originate close to one end of the dipole. In order to obtain some figure which represents the total discrimination of the microphone against all distant sounds, we can integrate the energy which the dipole microphone would receive from all directions; do the same thing for a non-directional pressure microphone; then compare the results. To make this comparison, it must be assumed, of course, that the two types of microphones have the same close-up sensitivity. An expression analogous to this comparison has been worked out for radio broadcast type microphones several times before,[2] and has been called the random energy efficiency. The random energy efficiency is given by the expression:

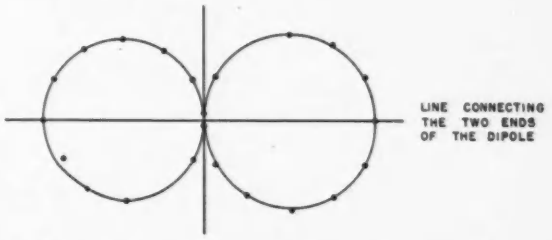

FIG. 7. Directional characteristics of an experimental dipole microphone measured at 1000 c.p.s.

$$\text{R.E.} = \tfrac{1}{2} \int_0^{\pi} f^2(\theta) \sin\theta d\theta, \tag{10}$$

[2] W. P. Mason and R. N. Marshall, J. Acous. Soc. Am. **10**, 214–215 (1938), Appendix A. Benjamin Baumzweiger, J. Acous. Soc. Am. **11**, 477–479 (1940).

where $f(\theta)$ is the directivity of the microphone expressed as a function of a plane angle θ. For conventional microphones, $f(\theta)$ is defined in terms of the maximum response of the microphone to plane waves; but since the dipole microphone is intended to discriminate against plane waves, we must redefine $f(\theta)$ in terms of

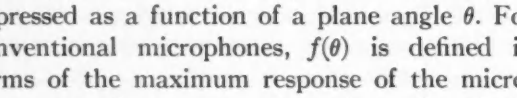

FIG. 8. Random energy efficiency of a close talking dipole microphone.

the close-up response of the dipole microphone. For the dipole microphone

$$f(\theta)=2\sin\frac{kD\cos\theta}{2}, \tag{11}$$

and

$$\text{R.E.}=2\left(1-\frac{\sin kD}{kD}\right). \tag{12}$$

For small values of kD, Eq. (12) reduces to

$$\text{R.E.}=\frac{(kD)^2}{3}. \tag{13}$$

Equations (12) and (13), then, represent the ratio of the random energy picked up by an ideal dipole microphone in a free field to the energy picked up by a non-directional pressure microphone, provided the two microphones have the same close-up sensitivity. Equation (12) is plotted in terms of db in Fig. 8. For $kD \leqq 1.9$, there is no discrimination against distant sounds. If the dipole spacing is 2.3 cm, the corresponding frequency would be 4500 c.p.s.

Since a completely analytical approach to a problem is likely to involve some rather dubious assumptions, an experimental check of the noise discriminating properties of the dipole microphone seemed desirable. There was some conflicting philosophy as to just what our experiments were intended to measure. That is, were we trying to compare the performance of microphones which might have widely differing frequency characteristics, and which were intended to be used at a variety of distances from the mouth? Or, were we to isolate the effect of using a dipole pick-up device? Both the construction of the dipole microphone and the form the analysis had taken led us to adopt the second point of view. To convert the dipole microphone into a non-directional pressure microphone, it was necessary merely to substitute for one of the original tubes another tube of the same diameter which was long enough to be acoustically "infinite." Such a change would affect neither the close-up sensitivity nor the frequency response, so we decided to measure the response of both the normal and converted microphones in a random noise field.

The microphone to be tested was set up in a large irregular concrete room which had a reverberation time of about 13 seconds. Amplified thermal noise reproduced by a loudspeaker was used to supply the random noise, and the output of the microphone was measured over a series of octave bands. The loudspeaker and microphone were oriented so that, as far as the horiontal plane was concerned, there were no detectable directional effects at the microphone position. To check the possibility of noise entering the microphone through the walls of the infinite tube, two infinite tubes were substituted for the dipole. The output of the microphone with the two infinite tubes was recorded as "leakage."

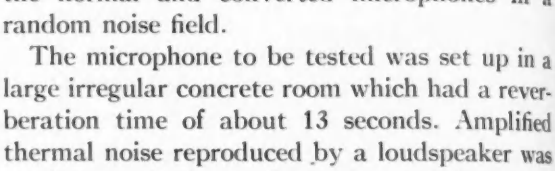

FIG. 9. Ambient noise response for dipole and pressure microphones.

Further checks seemed to indicate that most of the "leakage" noise was reaching the microphone diaphragm through the case walls rather than through the tube walls. Figure 9 shows the

results of the noise discrimination measurements. There is, in general, a trend toward a greater degree of discrimination for low frequencies, though the discrimination is not so great as that predicted analytically. The 3200–6400 c.p.s. band shows no discrimination at all, which agrees pretty well with the zero discrimination frequency of 4500 c.p.s. calculated for this same model.

Since the noise discrimination measurement was carried out under free field conditions, there is a legitimate objection that the test conditions were not representative of the conditions existing in actual use. Diffraction around the wearer's head had been ignored in the analysis; so as to evaluate tentatively the effects of diffraction, the experiment described above was repeated with a person wearing the microphone in the manner shown in Fig. 1. Although diffraction effects were noticeable, they were never more than one or two db.

In the foregoing analysis, we have assumed that the dipole was located very close to the source of the desired sound. It is a matter of practical interest, however, to determine the loss in effective sensitivity when the dipole is so far from the source that we can no longer neglect the pressure at one of its ends. If r is on the same order of magnitude as D, neither Eq. (8) nor Eq. (9) is applicable. Let us look once more at Fig. 6. If $\theta=0$ we can divide the absolute value of Δp from Eq. (7) by the absolute value of p_A from Eq. (5) and obtain, eventually, an expression which represents an effective loss in sensitivity.

From (5) and (7) if $\theta=0$,

$$\frac{|\Delta p|}{|p|}=\left[1-\frac{2r/D}{r/D+1}\cos kD+\left(\frac{r/D}{r/D+1}\right)^2\right]^{\frac{1}{2}}. \quad (14)$$

Figure 10 shows Eq. (14) plotted in terms of db for several values of kD and for r/D as the independent variable. If, for example, the dipole should be so far from the source that $r/D=0.5$, the effective loss in sensitivity would be about 3.5 db for most of the voice frequency range. Of course, the loss is greater for low frequencies than for high.

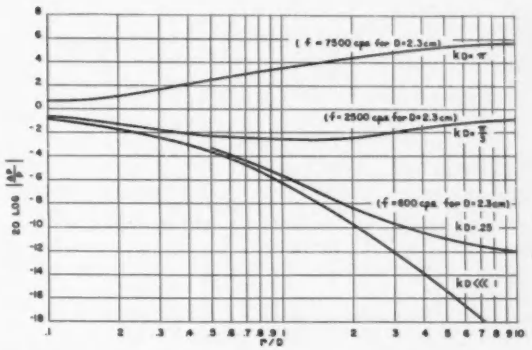

FIG. 10. Relative sensitivity as a function of the distance from the source and of the dipole spacing.

OTHER APPLICATIONS

So far we have mentioned only one application of the dipole microphone employing sound transmission tubes. If it should be desirable, for example in a distant talking microphone, the directional characteristics could be changed from the cosine type to cardioid by changing the length of one tube by an amount equal to the dipole spacing.[3] It might even be possible to construct a microphone in which one tube had an adjustable crook, like a trumpet tuning slide, and thus obtain a variety of directional patterns.

The small size of the tubes which can be used suggests the additional possibility of applying some form of tube microphone to an acoustic wattmeter, or an impedance measuring device, where it is desirable to make measurements without disturbing the sound field. The velocity component could be measured with some form of the dipole microphone, while the pressure measuring element could be a single-tube microphone with the open end of the tube terminated in the appropriate resistance.

[3] H. F. Olson, *Elements of Acoustical Engineering* (D. Van Nostrand Company, Inc., New York, 1940), p. 210.

A Wide Range Adjustable Acoustic Impedance

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IN connection with some experimental studies of sound fields, an adjustable acoustic impedance, capable of producing a wide range of impedance values over a small area (such as a 1″ diameter circle), was needed. A few variable acoustic impedances have been described in the literature.[1–3] These utilize acoustic absorption material and must be calibrated by direct measurement. Furthermore, the range of impedances obtainable was too limited for the intended use. Because of the difficulty in making acoustic impedance measurements, it was felt that the input impedance of any such device should be directly calculable from its dimensions. Such a device has been developed.

DESCRIPTION

The construction of the wide range adjustable acoustic impedance is shown in Fig. 1. It consists

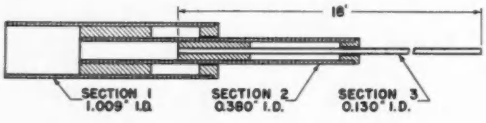

FIG. 1. Adjustable acoustic impedance.

of three sections of cylindrical tubing of different diameters. Sections *1* and *2* are metal; section *3* is an 18′ length of plastic (Saran) tubing. Section *3* has an attached collar which slides within section *2* so that the length of section *2* may be varied, maintaining an acoustically tight connection. Section *2* has a similar collar which slides within section *1*. The length of section *3* is sufficient to form a substantially infinite acoustic line, the input impedance of which affords a convenient, calculable termination for section *2*.

THEORY

In order to explain the operation of the adjustable acoustic impedance, we must first consider the input impedance of a cylindrical tube terminated by a general impedance as shown in Fig. 2. Mason[4] has given a solution of the

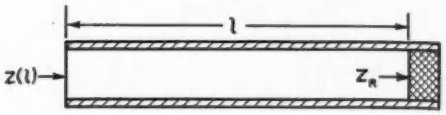

FIG. 2.

differential equation stated by Rayleigh[5] for plane waves in a tube, which corresponds to the equations for electrical transmission lines. This solution, if applied to the condition in which we are interested, results in the relation

$$Z(l)=Z_0\frac{Z_R\cosh\gamma l+Z_0\sinh\gamma l}{Z_0\cosh\gamma l+Z_R\sinh\gamma l}, \qquad (1)$$

where $Z(l)$ = the input impedance, Z_R = the terminating impedance, Z_0 = the characteristic impedance of the tube, $\gamma=\alpha+j\beta$ = the propagation constant, α = the attenuation constant, β = the phase constant, and l = the length of the tube. All of the impedances referred to are acoustic impedances.

The propagation constant is determined from

$$\gamma=\left\{-\frac{\omega^2}{c^2}\left[1+\frac{2}{r}\left(\frac{\gamma'^2}{2\omega\rho}\right)^{\frac{1}{2}}-j\frac{2}{r}\left(\frac{\gamma'^2}{2\omega\rho}\right)^{\frac{1}{2}}\right]\right\}^{\frac{1}{2}}, \qquad (2)$$

where $\omega=2\pi f$, f = frequency, r = radius of the tube, c = velocity of sound, ρ = density of the medium, $\gamma'=(\mu/\rho)^{\frac{1}{2}}+(\sqrt{\gamma_S}-1/\sqrt{\gamma_S})\sqrt{\nu}$, μ = coefficient of viscosity of the medium, γ_S = ratio of specific heats, and ν = coefficient of heat conductivity of the medium. If $2(\gamma'^2/2\omega\rho)^{\frac{1}{2}}/r$ is small, Eq. (2) becomes

$$\gamma=\alpha+j\beta\doteq\frac{1}{rc}\left(\frac{\omega\gamma'^2}{2\rho}\right)^{\frac{1}{2}}+j\left[\frac{\omega}{c}+\frac{1}{rc}\left(\frac{\omega\gamma'^2}{2\rho}\right)^{\frac{1}{2}}\right] \qquad (3)$$

$$=\alpha+j\left(\frac{2\pi}{\lambda}+\alpha\right),$$

[1] K. Schuster and W. Stohr (review), J. Acous. Soc. Am. **11**, 492 (1940).
[2] R. H. Brown and R. L. Bolt, J. Acous. Soc. Am. **12**, 31 (1940).
[3] E. C. Jordan, J. Acous. Soc. Am. **13**, 8 (1941).
[4] W. P. Mason, Bell Sys. Tech. J. **6**, 258 (1927).
[5] Lord Rayleigh, *Theory of Sound* (Macmillan and Company, Ltd., London, 1926), Vol. II, Sections 347–348.

where λ is the wave-length of sound in free space and

$$\alpha=\frac{1}{rc}\left(\frac{\omega\gamma'^2}{2\rho}\right)^{\frac{1}{2}}=\frac{1}{rc}\left(\frac{\pi\gamma'^2 f}{\rho}\right)^{\frac{1}{2}}.$$

Fay[6] has found that the measured attenuation constant in a tube comprises a component proportional to the first power of the frequency in addition to that given by Eq. (3). While this component proportional to the first power of the frequency is small in the case reported [about 6 percent of that given by Eq. (3) for a 0.749" I.D. tube], its dependency upon the tube dimensions is not yet known, consequently, the value of the attenuation constant is somewhat uncertain. Other workers[1,7] have reported values of the attenuation constant about 15 percent greater than that given by Eq. (3).

The characteristic impedance is given by

$$Z_0=\frac{\rho c}{S}\left(\frac{\gamma}{j\omega}\right)=\frac{\rho c}{S}\left[1+\frac{1}{r}\left(\frac{\gamma'^2}{2\omega\rho}\right)^{\frac{1}{2}}-j\frac{1}{r}\left(\frac{\gamma'^2}{2\omega\rho}\right)^{\frac{1}{2}}\right], \quad (4)$$

where S = cross-sectional area of the tube.

Equation (1) is of the same form as the equation for the input impedance of a length of electrical transmission line. Because of the analogous nature of the two cases, we are free to apply to the acoustic line the mathematical treatment customarily applied to electrical lines.

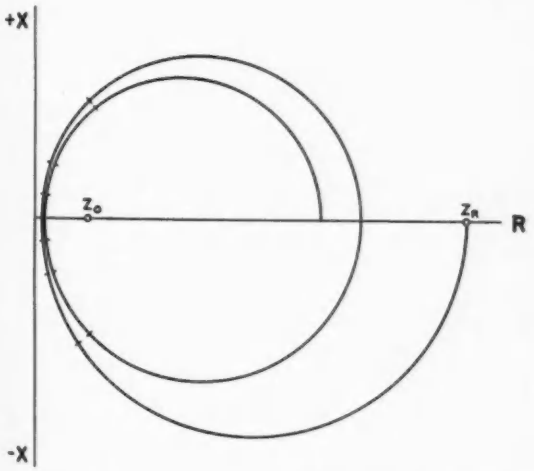

FIG. 3. Resistance-reactance plot of $Z_0 \tanh(\gamma l+\gamma_R)$.

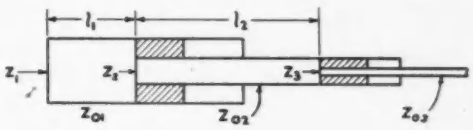

FIG. 4.

If we define a quantity

$$\gamma_R=\alpha_R+j\beta_R=\tanh^{-1}\frac{Z_R}{Z_0}, \quad (5)$$

Eq. (1) reduces to

$$Z(l)=Z_0\tanh(\gamma l+\gamma_R)$$
$$=Z_0\tanh[(\alpha l+\alpha_R)+j(\beta l+\beta_R)]. \quad (6)$$

The relation [Eq. (6)] between the input impedance $Z(l)$ and the length of the tube can best be described by a resistance-reactance plot of the input impedance as the length is increased from zero. Such a plot has been shown[8] to be a spiral, starting at the point representing the impedance of the termination (Z_R) and approaching the point representing the characteristic impedance (Z_0) as the length of the tube is increased; this is shown in Fig. 3. The short bars indicate equal divisions of tube length.

Referring now to the diagram of the adjustable acoustic impedance shown in Fig. 4, we shall define the symbols to be used as follows: Z_1 = the input impedance of the device, Z_{01} = the characteristic impedance of section *1*, l_1 = the length of section *1*, $\gamma_1=\alpha_1+j\beta_1$ = the propagation constant for section *1*, Z_2 = the impedance terminating section *1* (also the input impedance of section *2*), Z_{02} = the characteristic impedance of section *2*, l_2 = the length of section *2*, $\gamma_2=\alpha_2+j\beta_2$ = the propagation constant for section *2*, Z_3 = the impedance terminating section *2*, Z_{03} = the characteristic impedance of section *3*, $\gamma_{R1}=\alpha_{R1}+j\beta_{R1}=\tanh^{-1}Z_2/Z_{01}$, and $\gamma_{R2}=\alpha_{R2}+j\beta_{R2}=\tanh^{-1}Z_3/Z_{02}$. From the simple case previously discussed, it will be seen that the input impedance Z_2 of section *2* is that of a tube terminated in the impedance Z_3 of the infinite line, and that a resistance-reactance plot of the input impedance

[6] R. D. Fay, J. Acous. Soc. Am. **12**, 62 (1940).
[7] L. L. Beranek, J. Acous. Soc. Am. **12**, 3 (1940).
[8] J. C. Slater, *Microwave Transmission* (McGraw-Hill Book Company, Inc., New York, 1942), p. 39.

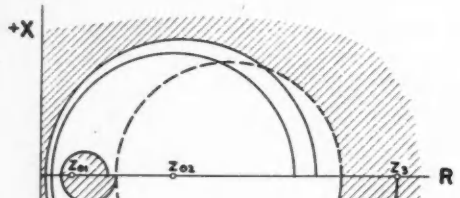

FIG. 5.

as a function of l_2 is a spiral. This is indicated by the dotted spiral of Fig. 5. Section *1* may likewise be considered as a simple tube terminated in the impedance Z_2. However, depending upon l_2, Z_2 may have any value lying on the dotted spiral. Hence the spiral representing the input impedance of section *1* as the length increases from zero, starts somewhere on the dotted spiral (depending upon the value of l_2) and progresses about the point Z_{01} representing the characteristic impedance of section *1*. Thus, a series of spirals is possible; that is, each setting of l_2 will determine a new spiral of input impedance. This is illustrated by the full line spirals of Fig. 5 which show the limiting cases. It is possible, then, to obtain all impedances lying within the unshaded area in Fig. 5 by adjustment of l_1 and l_2 between zero and $\lambda/2$. If the dotted spiral passes through the point Z_{01}, the small area of unobtainable impedances about Z_{01} vanishes.

The impedance terminating section *1* is

$$Z_2 = Z_{02} \tanh\left[\gamma_2 l_2 + \tanh^{-1}\frac{Z_3}{Z_{02}}\right]. \qquad (7)$$

The input impedance is then

$$Z_1 = Z_{01} \tanh\left[\gamma_1 l_1 + \tanh^{-1}\frac{Z_2}{Z_{01}}\right]$$

$$= Z_{01} \tanh\left\{\gamma_1 l_1 + \tanh^{-1}\left[\frac{Z_{02}}{Z_{01}} \tanh\left(\gamma_2 l_2 + \tanh^{-1}\frac{Z_3}{Z_{02}}\right)\right]\right\}. \qquad (8)$$

While it is not too difficult to calculate the input impedance for given settings of l_1 and l_2 by means of Eq. (8), because of the nature of the hyperbolic functions it is considerably more difficult to determine the settings of l_1 and l_2 for a required input impedance Z_1. The settings, however, may be determined more easily by a semigraphical method. Equation (8) may be rearranged as follows:

$$Z_{01} \tanh\left[\left(\tanh^{-1}\frac{Z_1}{Z_{01}}\right) - \gamma_1 l_1\right]$$

$$= Z_{02} \tanh\left[\left(\tanh^{-1}\frac{Z_3}{Z_{02}}\right) + \gamma_2 l_2\right]. \qquad (9)$$

The right-hand member of Eq. (9) is seen to be Z_2 [from Eq. (7)] and is shown in Fig. 6 by the dotted spiral. The left-hand member of Eq. (9) differs in form from the right-hand member only in the negative sign of $\gamma_1 l_1$. Thus it represents a spiral starting at Z_1 but progressing counterclockwise about Z_{01}. If points corresponding to lengths of l_1 and l_2 are marked off along the spirals, the values of l_1 and l_2 for the required input impedance may be taken directly from the curves, being determined by the point of intersection of the two spirals. Or, the lengths may be determined by equating the impedance repre-

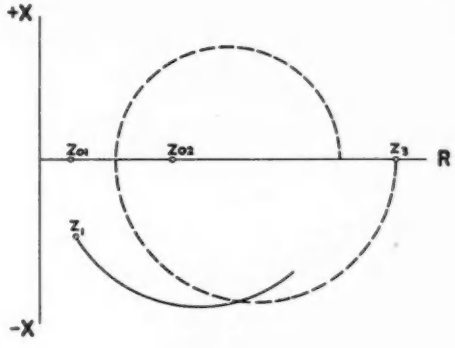

FIG. 6.

sented by the point of intersection of the two spirals to the left-hand member of Eq. (9) to find l_1 and to the right-hand member to find l_2.

The impedance Z_3 terminating section *2* need not be that of an infinite line but may be any known impedance. For some purposes, it may

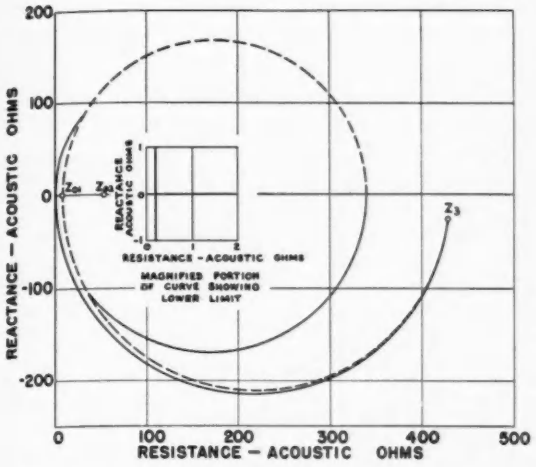

FIG. 7. Resistance-reactance plot showing range of the adjustable acoustic impedance of Fig. 1 (833 c.p.s.).

be necessary to know the pressure developed at the terminating impedance Z_3. In this case, a calibrated probe microphone of known impedance can be used as the terminating impedance. By means of the probe microphone, the pressure at the termination and, consequently, that at the input, may be determined.

It should be remembered that the preceding theory is based on Eq. (1) which assumes plane waves in the tubes. Such an assumption is probably valid when the change in area between sections is not too great, but a correction may be necessary when the change in area is large. We are not prepared to evaluate such a correction at this time.

It should also be noted that because of the analogous nature of electrical, acoustic, and mechanical transmission lines, the preceding relations may be adapted to any of these types of lines.

RANGE

The calculated range of the adjustable acoustic impedance having the dimensions given in Fig. 1 is shown in Fig. 7 for a frequency of 833 c.p.s. The range does not differ greatly for other frequencies. The calculations were based on the average of measured values of Z_3 and values of the attenuation coefficient calculated from

$$\alpha = \frac{2.80 \times 10^{-5} \sqrt{f}}{r} \text{ per cm},$$

corresponding to the real part of Eq. (3) for dry air and room temperature.[6]

It is seen that the range includes roughly all the impedances which lie within a circle having its diameter on the R axis, the end points of the diameter being $R = 0.02$ acoustic ohm and $R = 340$ acoustic ohms. Since the dotted spiral passes very nearly through the point Z_{01}, there is only a small region about Z_{01} in which lie unobtainable impedances.

MEASUREMENTS

Some measurements of the input impedance of the adjustable acoustic impedance were made by means of a modified transmission line method.[9] A tube having a small diameter (0.314″ I.D.) was used as the measuring line and a long probe microphone was used to explore the pressure in the measuring line as shown in Fig. 8. The probe tube, even though small, occupied a considerable portion of the cross-sectional area of the measuring line, consequently, movement of the probe tube affected the level of sound in the line. In order to eliminate this error, the pressure measured by the exploring microphone was compared with the pressure measured by a high impedance probe microphone (reference microphone) located near the termination of the measuring line. As the impedance of the reference microphone was about 50 times that of the highest impedance measured, the error introduced by this shunt impedance was considered negligible. Furthermore, any error resulting from the difference between the cross-sectional areas of the measuring line and the impedance being

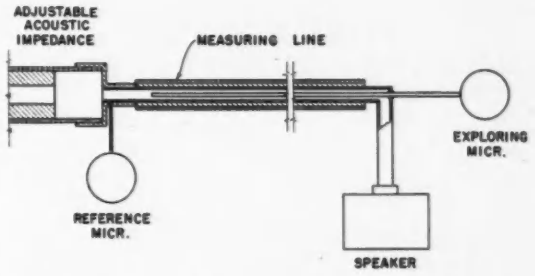

FIG. 8. Diagram of apparatus used for impedance measurements.

[9] W. M. Hall, J. Acous. Soc. Am. **11**, 140 (1939).

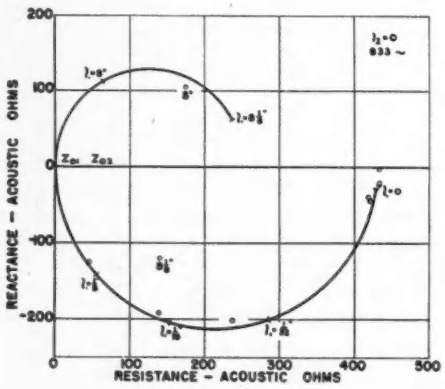

FIG. 9. Calculated and measured impedances for several settings of the adjustable acoustic impedance.

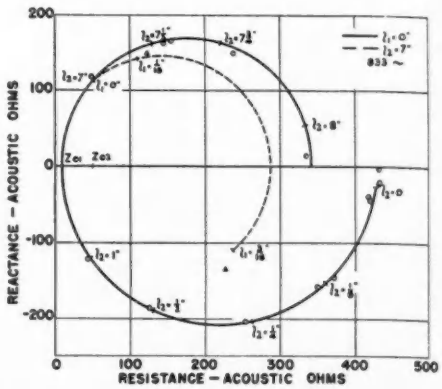

FIG. 10. Calculated and measured impedances for several settings of the adjustable acoustic impedance.

measured was neglected. No attempt was made to provide means for precision measurement of the section lengths; these were measured by an ordinary machinist's scale.

Figure 9 shows the comparison between calculated and measured values of input impedance when l_2 is held at zero and l_1 alone varied. The scattering of points about $l_1=0$ represents repeated measurements of Z_3, the impedance of the "infinite" section, and give an indication of the repeatability of the measurements. The cross-sectional area of the tubing used for the "infinite" section (section *3*) was not strictly uniform, hence its impedance varied with frequency and differed somewhat at the frequency of the measurements from the characteristic impedance calculated from the nominal dimensions of the tube. Consequently, the average of the measured values of Z_3 was used in the calculations. The temperature was observed throughout the measurements and the calculated values of input impedance were corrected for slight temperature changes which occurred during the course of the measurements. The temperature correction was applied only to the determination of λ in Eq. (3) and was calculated from the relation

$$\lambda=(33050+60.7\times t°\text{C})/f \text{ centimeters.}$$

There is seen to be fair agreement between measured and calculated results for small values of l_1. For values of l_1 in the vicinity of 8″ (corresponding to about $\lambda/2$ at 833 c.p.s.) there are large discrepancies. We are unable at present to say whether these discrepancies are attributable to the abrupt changes in area at the junctions between sections or to other causes. It will be noted, however, that most of the range may be covered with lengths of l_1 less than $\lambda/4$.

Figure 10 shows the comparison between measured and calculated values of input impedance for two conditions. In the first case, l_1 was held at zero and l_2 varied; in the second, l_2 was held at 7″ and l_1 varied. It is seen that there is good agreement between measured and calculated values in both cases, the difference being less than 8 percent in magnitude and 6° in phase angle.

DISCUSSION

The adjustable acoustic impedance, while promising to be a useful device, is not without its disadvantages and difficulties. In the first place, the input impedance must be calculated in each case (unless eventually some calibration chart can be devised), and the calculations, although simplified by the use of the Kennelly charts,[10] require considerable time. Furthermore, in certain regions the input impedance is very sensitive to frequency and temperature changes. Where great accuracy is required, such as in the use of the device as the standard impedance of an acoustic impedance bridge, the error introduced by the abrupt changes in area should be evaluated and the propagation constant should be more accurately determined.

[10] A. E. Kennelly, *Chart Atlas of Hyperbolic and Circular Functions* (Harvard University Press, Cambridge, 1927).

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Some Applications of Ultrasonics in High-Polymer Research

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1. THE SUPERATION OF SURFACE TENSION

IT has been known, since long ago, that sound waves and ultrasonics can profoundly affect colloidal systems.[1–5, 7, 8, 11, 12, 24, 25] K. Sollner[20] in a recent article has given an excellent survey as to how stable emulsions can be produced under certain conditions and how eventually they can be destroyed under others. The degree of dispersion in these cases is mostly moderate with average particle diameters ranging from one to twenty or more microns; the forces which establish dispersion equilibrium are of the nature of surface tensions and of the order of magnitude of about 100 to 1000 cal. per mole of the dispersed phase. It is not too difficult to understand that standing ultrasonic waves in a liquid system having wave-lengths between 100 and 1000 microns can produce or destroy suspensions of this comparatively coarse nature being established by such weak forces.

It seems, however, that under favorable conditions ultrasonic waves can produce much deeper going changes of chemical systems. They can affect not only intermolecular forces of the van der Waals type (1000–10,000 cal. per mole), but are even capable of disrupting strong chemical bonds (50,000–100,000 cal. per mole). This review attempts to give a short report about a few outstanding facts and their possible interpretation.

2. THE OPENING OF VAN DER WAALS BONDS

If one subjects concentrated solutions of high polymers such as a 5 to 10 percent solution of gelatine in water or a similar solution of rubber in toluene, to the action of an ultrasonic field (about 20,000–100,000 cycles) of moderate intensity (about 10–20 watts per sq. cm.) it can be observed that after a short time the apparent viscosity of the system has markedly decreased. This viscosity decrease, however, is not final and irreversible, because if the irradiation of the solution is interrupted, the system "recovers" and, after a certain time, reassumes its initial viscosity or a value near to it. The discoverers of this effect[4–6, 10, 19, 23] have interpreted it as follows: High-polymer solutions of the kind under consideration do not possess viscosity as defined by the fundamental laws of Newton, Poiseuille, or Stokes.[9] Their long and irregularly entangled chain-like molecules form a flexible framework, inside of which, as in a loose sponge, the solvent is entrapped and immobilized. Hence the system offers considerable resistance to the action of shearing forces and, as long as these forces are too weak to tear the network to pieces, appears to have a high viscosity, or even has the nature of a gel. If one sufficiently increases the shearing forces, however, the framework is destroyed and the system becomes fluid. This can can be done by increasing the pressure difference at the ends of a capillary through which the solution flows, as shown in the fundamental experiments of Bingham* or by shaking of the gelatinous ("thixotropic") solution[5, 20] or by stirring and other mechanical agitation. It seems that the viscosity decrease of such system under the influence of sound or ultrasonics is also caused by the fact that the standing waves somehow destroy the network of the long chain molecules. According to Freundlich and Gillings[4] the destruction of the irregular network of the solution is not immediately effected by the waves themselves, but is a consequence of a "cavitation" effect caused by the existence of dissolved air in the solution, which forms little caves as the system is exposed to irradiation. These caves expand and contract and hereby disrupt the continuous network of the macromolecules. In this sense, the effect is some sort of microscopical shaking which severs the bonds between different chain molecules. These bonds are established between the chains by comparatively weak intermolecular (van der Waals) forces as shown by the fact that they can also be broken by simple

* Cf. reference 9.

FIG. 1.

shaking or stirring. The energy equivalent of such intermolecular bonds is between 2000 and 8000 calories per mole of the binding groups.

A direct experimental evidence for the importance of cavitation is found in the influence of a pressure superimposed on the ultrasonic field. If one irradiates at 10 atmospheres, cavitation is greatly diminished and the effect of the ultrasonic field on the viscosity is nearly unobservable.

After irradiation has been stopped, the macromolecules, under the influence of their irregular Brownian movement, slowly reproduce the random network in the solution and the original viscosity of the system is re-established.

It seems, therefore, that the *temporary viscosity decrease* of high-polymer solutions in an ultrasonic field of moderate intensity can be readily understood as the *temporary opening* of *van der Waals bonds*, the moderate strength of which should not offer insurmountable resistance to the ultrasonic vibrations.

3. THE BREAKING OF CHEMICAL BONDS

Recently it has been observed[13–17,23] that, under certain conditions, *irreversible* viscosity decreases can be effected in high-polymer solutions. Using frequencies around 300,000 cycles and intensities around 10 watts per sq. cm, Schmid and his collaborators[13–17] have observed that solutions of nitrocellulose, polyvinyl acetate, and polystyrene, if irradiated for a few hours at about 70°C, show a considerable viscosity decrease, which does not recover, even if the solution is completely evaporated and the dry polymer redissolved in the solvent.

The following figures intend to give an impression of how this phenomenon is affected by various experimental conditions. The viscosities η_c of the solutions as measured after different periods have been converted into specific viscosities η_{sp} [9] according to

$$\frac{\eta_c}{\eta_0} - 1 = \eta_r - 1 = \eta_{sp}, \tag{1}$$

η_0 = viscosity of the pure solvent.

The specific viscosities, again, have been converted into "viscosity average molecular weights" MW with the aid of the Staudinger equation[9]

$$MW = \eta_{sp} \times C, \tag{2}$$

where C is an empirical constant, characteristic for each system solvent-solute at very low concentration.† It is recognized that Eq. (2) is only a rough approximation but is presumably good enough to reflect the general character of the phenomenon and will be used as long as the molecular weights do not exceed 300,000. It interprets irreversible viscosity drops as a molecular weight decrease and hence as a chemical degradation.

TABLE I. Forces corresponding to some chemical bonds.

Bond	Force in 10^{-4} dyne per bond
C—C	4.50
C—O	5.77
C=C	9.77
C=N	12.06
C≡N	16.6
C≡C	17.2

Figure 1 shows the degradation of three polystyrene solutions at 70°C in toluene. The initial MW were 100,000, 150,000, and 300,000, the corresponding MW after 2 hours were 30,000, 70,000, and 40,000. It can be seen that in all cases the major part of the degradation was accomplished after 20 minutes. After that time the material was much more resistant against the

† Cf. references 16, 17 for numerical values.

waves than before. The sample having an initial *MW* of 150,000 is more stable than the two other samples, the one with the initial *MW* of 300,000 is very susceptible to degradation. Hence it seems that different polystyrene samples, although being considered as chemically very similar, can behave differently towards the influence of ultrasonics. This corroborates other evidence, indicating that there exist configurational differences in high-polymer molecules, even if their chemical composition is identical and even if their average molecular weight is of the same magnitude.

It was found that the final samples had a much narrower molecular weight distribution than the original ones. This indicates that ultrasonics might be used to carry out a well-controlled degradation of an inhomogeneous initial polymer into a more homogeneous end-product.

Figure 2 shows the influence of the intensity of the field and of temperature. The relative viscosity

$$\eta_r = \eta_{sp} + 1 \qquad (3)$$

is here plotted *versus* the intensity of the field in watts per sq. cm. The initial *MW* was 300,000; two runs were made at 70°C and 80.5°C, respectively; the time of irradiation for all experiments was 90 minutes. It can be seen that 10 watts per sq. cm represent some sort of saturation intensity for the chosen experimental conditions.

Figure 3 finally represents the influence of external pressure on the irreversible viscosity

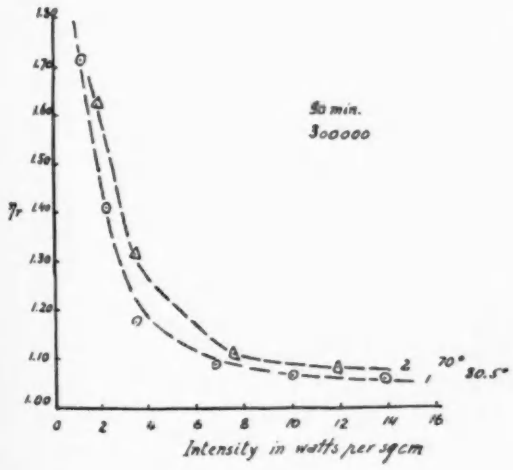

FIG. 2.

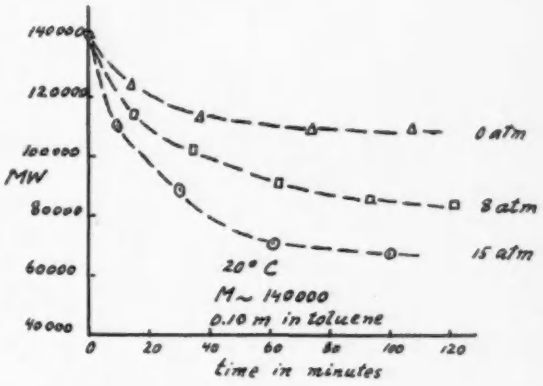

FIG. 3.

decrease. One and the same fraction, having an initial *MW* of 140,000, was irradiated with about 300,000 cycles and 10 watts per sq. cm at 20°C in a 0.10 molar solution in toluene at three different external pressures (no pressure, 8 and 15 atmospheres). Additional external pressure increases very markedly the degrading influence of the ultrasonic field and shows that, in this case, there can be no effect of cavitation.

Everything, therefore, points to the fact that macromolecules, having molecular weights around and above 100,000, can be chemically degraded by ultrasonic waves. In order to understand this effect, we have first to ask ourselves: (1) which force is necessary to break a covalent chemical bond, and (2) what is the average size and shape of such macromolecules in solution?

Spectroscopy, particularly infra-red absorption, has permitted the computation of the potential energy curves for the most important covalent bonds and therefrom to derive the force necessary to open the bond. Table I gives a number of representative values and shows that, for the C−C bond, as it occurs in polystyrene or the C−O bond, as it occurs in nitrocellulose, the force is about 5×10^{-5} dyne per bond.

As to the average shape of macromolecules in dilute solution, recent investigations of Simha,[18] Gralen,[6] and others[9] indicate that they are neither completely stretched out, nor completely coiled up, but show an intermediate shape of moderately undulated configurations. Figure 4 sketches the situation. A polystyrene molecule with a molecular weight of 100,000 would have a length of 3000A and a diameter of 6A if it were

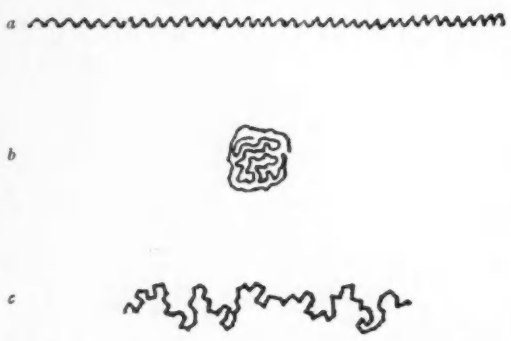

FIG. 4. *a*, $l/d = 50$, stretched-out configuration. *b*, $l/d = 1.0$, curled-up configuration. *c*, $l/d = 10$, intermediate configuration.

completely stretched out. Its axis ratio *f* would be 500. If the same molecule were coiled up to a little solid polystyrene sphere, its diameter would be 75A, and its axis ratio, of course, would be one. It seems that none of these two extremes is actually materialized. Much rather the average shape of such a chain seems to be the slightly curved configuration as shown in Fig. 4 at the bottom, with an average effective length of about 1500A and an average effective diameter of about 15A, having thus an axis ratio of about 100. In a dilute solution these molecules are carrying out irregular internal vibrations and rotations and also move as a whole slowly through the solution. The internal movement (micro-Brownian movement according to W. Kuhn) is comparatively fast. Small segments of the chains are changing their local configuration within about 10^{-3} second. The molecules as a whole are moving slowly (macro-Brownian movement) through the solution; their average velocity is about 0.5–1.0 micron per second.

These molecules are now exposed to a standing ultrasonic wave of an intensity around 10 watts per sq. cm, a frequency of 3×10^5, a velocity of propagation of about 1000 m per second, and a wave-length of 0.5 cm. Figuring out[14] the maximum velocity which the solvent molecules acquire (on top of their regular Brownian motion) through the influence of the wave, one finds about 40 cm per second. If one assumes that the inertia of the high-polymer molecules is so large that the molecules are not completely swept away by the liquid movement of the wave, but resist this movement, then a friction will be set up between the molecules and the liquid. The friction can be computed from the viscosity of the liquid (about 10^{-2} poise), the dimensions of the molecule, and the relative velocity of the flowing solvent to the macromolecule. Using the shape as indicated in Fig. 4*c* and the above figures for solvent motion and solvent viscosity, one arrives at a frictional force of about $2\text{–}3 \times 10^{-4}$ degree. Comparing this with Table I, we see that in fact the maximum possible frictional forces between the macromolecules and the solvent are about five times larger as required to break a C—C or C—O bond. If they would really act fully, the polymer would be completely disintegrated as soon as the ultrasonic field is switched in. This, of course, is not the case because the inertia of a single macromolecule is not large enough to resist the flow of the liquid completely. On the contrary, most of the chain molecules will join the vibrational motion of the solvent and only rarely, here and there, will considerable friction develop. This explains the very low degree of degradation as represented in Fig. 1 (only five bonds in 1000 are broken), and the long time it takes to achieve even this small cleavage. It also explains why the efficiency of the degradation drops so rapidly with decreasing molecular weight because the accumulated friction depends sharply upon the effective length of the molecule. Finally it may be added that the influence of internal pressure can be reasonably explained by the fact that the viscosity of the liquid is hereby increased.

Thus it seems that the very interesting effect of a chemical degradation of high-polymer molecules by the mechanical[21,22] action of ultrasonic waves can be fairly well explained by our present knowledge of the strength of covalent bonds and the structure of high-polymer molecules.

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Intonation of the Boehm Clarinet

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THERE are grave faults in the intonations of the wind instruments and especially the wood winds. There has been very little improvement in the intonation of the Boehm clarinet in nearly a hundred years. The faults are not of merely academic interest but are so appreciable as to discourage the more universal use of an instrument which excels in beautiful tone quality and dynamic flexibility. We have recorded the intonations of many clarinets and have found that the results may be expressed in typical form. That part of the pitch range most involved in the faulty tuning has been divided for study purposes into two series of tones separated by the interval of an equally tempered twelfth. All frequencies were measured by the application of Lissajous figures using standard tuning forks with the clarinet tones. It was found that, on producing any clarinet tone, a steady state for frequency required considerable time. We have suggested the universal use of a normal temperature as a requirement in manufacturing for optimum intonation. It is also shown that better intonation is attained by leaving the barrel adjustment at that which corresponds to normal temperature at all times rather than attempting to change the barrel position for different temperatures. Since most of the faults are inherent in the present mode of construction of the clarinet, there must be some radical alteration to correct the intonation. This is found to be possible and, in fact, actually to have been previously suggested and carried out practically by Mr. C. E. Potter in 1926. The improvement involves no new fingering.

INTRODUCTION

Clarinets, like all other wood-wind instruments, are considerably out of tune, perhaps played out of tune, but most certainly made out of tune. This fault is inherent in the make-up of the instrument, that is, the shape and size of its bore, the placement and dressing of its tone holes, and the rise of the pads above the tone holes when the key is manipulated by the finger.

It must not be supposed that this faulty intonation is a purely incidental matter. It is present in all makes of clarinets, so far as numerous tests have shown, and the tonal error varies on different notes from zero to a full quarter-tone—or even more in the case of an unskilled player with faulty embouchure.

The difficulty may be partially overcome by a skillful player who may temporarily alter the tube length or change the embouchure; but to accomplish this, the attention must be diverted from musical interpretation where the greatest concentration is desirable. The wood-wind choir of every symphony orchestra seethes with suspicion as to the source of faulty intonation. See Fig. 1.

FIG. 1. The Boehm system clarinet, 17 keys, 6 rings. *1*. Mouthpiece. *2*. Adjustable barrel joint. *3*. Joint for left hand. *4*. Joint for right hand. *5*. Bell joint.

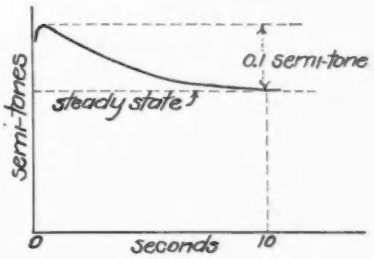

FIG. 2. Pitch transition to the steady state.

The intonation of a clarinet, that is, the pitch of each tone when played by a skilled performer, is a function of tube length, placement, and dressing of tone holes, and temperature within the tube. The tube length may be altered by adjustment of the barrel joint which is located next to the mouthpiece.

It must be assumed that there should be a normal or standard temperature at which the clarinet is designed to have its optimum intonation. But there is no experimental evidence that manufacturers have agreed upon any such standard, normal temperature. A reasonable normal temperature would be 68°F or 20°C. At this temperature even a most favorable placement of the tone holes and drawing of the barrel does not produce correct intonation with the system as it is; in fact, the intonation is generally not acceptable.

THE STEADY FREQUENCY STATE

The production of any tone on the clarinet is accompanied by a transitional frequency change which appears thus far to have escaped notice. The initial sound is too sharp by about 0.1 semitone and about 10 seconds is required to establish a steady state. Obviously the pitch of most tones in a musical rendition never reaches a steady state. This is true in all registers. Figure 2 is drawn to show the effect.

Probably (the natural frequency of the reed being higher) the frequency of the tone rises in some negligible time, then, having been brought under the influence of the air column, falls during the ten seconds. At the initiation of the standard orchestral tuning tone of 440 cycles per second there are about two beats per second with the standard tuning fork. These disappear as the correct steady pitch is assumed. Since shorter notes are more frequent in music it appears that they should be used for tuning. This effect explains in part the difficulty of playing the clarinet with an instrument of fixed intonation such as a piano. Our measurements were all taken after the frequency and pitch had reached the steady state.

PITCH NOTATION, RANGE, AND INTONATION

"Middle C" is notated[1] as C_4 and the useful range of the clarinet in $B^\flat$ extends from E_3 to G_6 as read by the performer, the actual sounds being two semitones below the notation. In this article the notes are indicated first as read by the performer and then, in parenthesis, as actually sounded. All results refer to the $B^\flat$ clarinet unless otherwise specified.

The three customary divisions[2] in pitch range are $E_3(D_3)$ to $B_4{}^\flat(A_4{}^\flat)$, $B_4(A_4)$ to $C_6(B_5{}^\flat)$, and $C^\sharp_{\sharp 6}(B_5)$ to $G_6(F_6)$ corresponding, respectively, to low, middle, and high registers, but only the low or middle registers are of interest in a study of intonation as the problem of the high register may be regarded as solved when the other two are correct.

For intonation study the low and middle registers have been subdivided into three series of tones which are referred to as series I, $E_3(D_3)$ to $F_4(D^\sharp_{\sharp 4})$, throat tones $F^\sharp_{\sharp 4}(E_4)$ to $B_4{}^\flat(G^\sharp_{\sharp 4})$, and series II, $B_4(A_4)$ to $C_6(B_5{}^\flat)$. See Fig. 3.

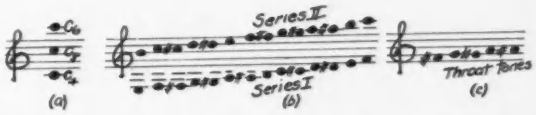

FIG. 3. (*a*) Octave notation. (*b*) Interval relations between series I of 14 tones (E_3 to F_4) and series II of 14 tones (B_4 to C_6). (*c*) Connection tones between series I and series II. Throat tones.

Series I and series II are closely related because when any given tone in series I is produced, a tone in series II a twelfth above may be sounded by opening the register hole with the key for the left thumb. The throat tones are produced nearly independently of the tones in the two series so that these holes may be correctly placed.

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Thus, it is evident that even if the tone holes for series I and also those producing the throat tones were correctly placed, still, correct intonation would require that each of the fourteen tones of series II be pitched a correct twelfth above the corresponding tone of series I as shown in Fig. 3. Each of these latter adjustments must result from the opening of the same register hole by its key. Numerous tests have shown that this ideal condition of correct twelfths is not attained in practice and so the twelfths as measured are referred to as contracted or expanded by an amount stated as a fraction of an equally tempered semitone.

Figure 4 represents two extreme cases between

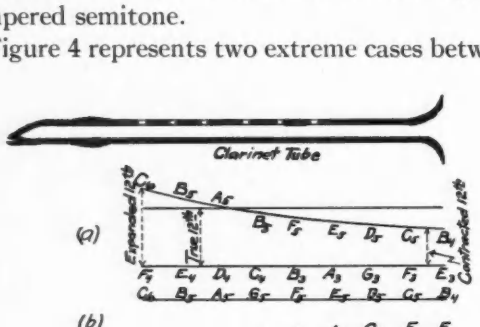

FIG. 4. (*a*) Series II is in error as shown. Series I is made correct by construction provided one of these is tuned correctly as based upon 440 cycles per second for B_4. (*b*) Series I is in error as shown. Series II is made correct by construction provided one of these tones is tuned correctly, i.e., based upon B_4 as 440 cycles per second.

which the instrument maker may influence the intonation by variations in placing and dressing the tone holes. The error measured in parts of an equally tempered semitone is seen to be a function of the effective tube length, i.e., the distance from the reed[3] to the first open tone hole. The relative position of the first open tone hole is indicated by the proper letter of the musical scale. The mouthpiece is on the left.

Also the absolute pitch, even after the tone holes are fixed, depends upon what tone is chosen for tuning to standard pitch. In Fig. 4(*a*), if $B_4(A_4)$ were chosen as the tuning note, then all tones except $B_4(A_4)$ would be somewhat sharp; for the distance between the parallel lines is the true twelfth, and must remain constant even when the frequencies of all tones in both series I and II are changed by adjusting the barrel. In the case shown, $B_4(A_4)$ is the poorest possible choice for a tuning note. Obviously $F_5(D\sharp_5)$, used as a tuning note, would distribute the error more equitably over the two series.

In Fig. 4(*b*), note $B_4(A_4)$, which is customarily used for tuning, would be fairly appropriate, but not as good as $B_3(A_3)$ which would distribute the error more equitably.

Evidently, errors of intonation are inherent in all clarinets for they are all patterned after a conventional design which has not been basically altered for over a hundred years. It is small wonder that amateurs cannot manage to play with pianos, and that professionals must display remarkable agility with the embouchure, fine pitch discrimination; and also be thoroughly familiar with temporary adjustments of the barrels of their particular instruments to play even reasonably well in tune. This is made all the more difficult because pitch is not disguised by vibrato (which is not naturally adaptable to the clarinet and not used by the best artists). Intonation is anything but a serious worry in modern jazz; but in the symphony orchestra and with chamber music it is always a burning issue in the wood-wind section, where the intonations of other instruments such as oboe and bassoon are as bad as or worse than that of the clarinet.

THE EFFECTS OF DIMENSIONS

The thicker the wall in the wood clarinet, or the longer the short lateral tube of a metal clarinet, the greater is the effective length of the resonating column. Hence, a tone may be made sharper by shortening this lateral tube leading to the outside air. Also a tone may be sharped by increasing the diameter of the tone hole. This latter partially removes the disturbing effect of that part of the air column below the resonating portion. Tone holes, especially the larger ones, may be enlarged by undercutting, that is, by making them larger at the point where they leave the bore. This improves the quality of tone but it must not be used to excess.

Increasing the diameter of the whole bore sharpens the pitch so that to maintain standard pitch the tube must be made longer. Also, in

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general, the larger the bore, the greater the difficulty in producing overtones.

The general effect of increasing the diameter at the bell end only is to raise the pitch while increasing the diameter only at points nearer the mouthpiece has the opposite effect. The effective bore may be made smaller by inserting a piece of wire into the clarinet.

TEMPERATURE EFFECTS

Clarinets are ordinarily played at temperatures varying, say, from 65° to 85°F. They may be tuned either by adjusting the length of the barrel joint until $B_4(A_4)$ sounds 440 cycles per second, or by leaving the joint at that length which would sound 440 cycles per second at normal temperature. The former method vitiates the correctness of the semitone intervals which were made to be as accurate as possible at the normal temperature. Also, only a few of the tones will be of correct pitch. This will be shown to be an inferior method.

The latter condition, where the length corresponds to optimum intonation at normal temperature, will cause the pitches of all tones to be sharp at a temperature higher than normal. In fact, the change in frequency will be about the same percentage or interval for every note. It may be seen that if a standard pitch correspondingly above normal could be used, a very favorable intonation would result. Thus, in the case of chamber music for strings and clarinet and played at any temperature, the strings should tune with the clarinet, the barrel of which is left in the position giving best intonation at normal temperature. In fact, the same arguments would probably apply to other wind instruments of the orchestra.

It is found necessary to "warm up" the clarinet before a performance but the resulting rise in pitch is caused in great measure by moistening and conditioning the reed.[3] This is shown by testing after soaking the reed in water.

EXPERIMENTAL PROCEDURE

For clarinets already constructed it is important to measure the equally tempered twelfths between corresponding tones of series I and II. The equally tempered twelfth interval is 2.9966 so that the difference between this and the just or natural twelfth which is 3.0000 would be only about 0.1 percent.

We used the principle of Lissajous figures to measure the frequencies of the tones in series I and II and thus the intervals between them. Tuning forks calibrated in the equally tempered scale were used as frequency standards. A Western Electric 13A beat frequency oscillator was connected to the horizontal deflection plates of an oscilloscope, and a piezoelectric or pressure microphone, Brush R22, to the vertical deflection plates. The fundamental of the clarinet tone was isolated by filter action of the degenerative type as is used in the General Radio harmonic analyzer.

Although the oscillator is frequency calibrated, we used this calibration only in obtaining small differences in frequency between the tuning fork and the frequency required to give a stationary Lissajous figure. For example, in testing the $D_4(C_4)$—$A_5(G_5)$ interval, the fork of frequency 261.6 was used. This is the correct frequency for D_4 while that of A_5 is 784.0. The oscillator is adjusted to give a stationary figure of 1:1 ratio of frequencies with the fork of frequency 261.6. Next, D_4 is sounded on the clarinet and the oscillator readjusted to a stationary 1:1 figure. The amount of frequency readjustment in cycles per second is the intonation error in the clarinet tone. Next, the A_5 is sounded on the clarinet and adjusted to a 3:1 stationary figure. If A_5 on the clarinet were 3 times 261.6 (784.8), the oscillator setting would be the same as for the fork on the 1:1 figure above. The frequency calculation for A_5 or the interval will run like this: Suppose the oscillator setting when the fork produced a 1:1 figure was 260; and the setting when A_5 on the clarinet produced a 3:1 figure was 261. The frequency of the oscillator at this setting is 261.6+1 or 262.6. The error in A_5 will be 787.8−784.0 or +3.8 cycles per second. If the lower tone D_4 were correct this would also be the error in the interval, although it is better to convert it to semitones by dividing by the size of a semitone in cycles per second at the frequency 784.0.

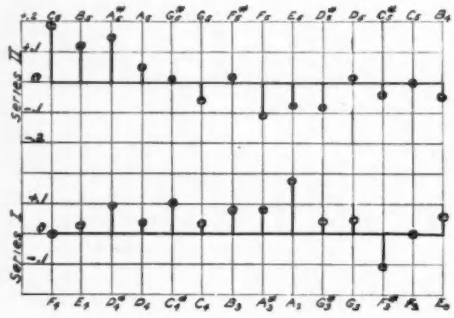

FIG. 5(a). Errors in tones of the two series in semitones. Clarinet No. 23454 at 82°F. Barrel joint pull is 2 mm.

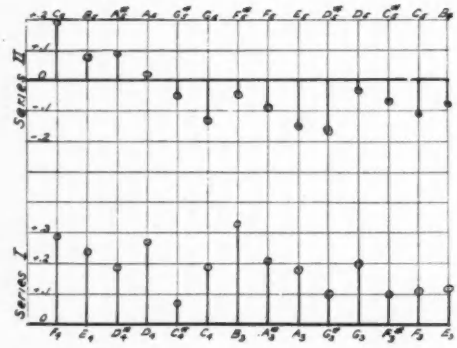

FIG. 5(b). Errors in tones of the two series in semitones. Clarinet No. 23454 at 71°F. No barrel pull. One would naturally expect good intonation under these two conditions.

EXPERIMENTAL RESULTS

Clarinet #23454 is a high grade instrument made in France; and its errors are typical of many clarinets examined, provided it be used at a room temperature of about 82°F. Figure 5(a) shows its intonation at this temperature. When used at 71°F, a very reasonable temperature, the intonation deteriorates considerably as in Fig. 5(b).

Considering that errors in clarinet intonation are often one- to four-tenths of a semitone, one-tenth of a semitone error is assumed allowable. Figure 5(a) shows that 82°F is a favorable temperature for the intonation of this clarinet because series I is practically correct; and series II is practically correct, except for $A\sharp_5(G\sharp_5)$, $B_5(A_5)$, and $C_6(A\sharp_5)$, with plus and minus errors equally distributed. Note also that the error in the contracted twelfth, $A_3(G_3)$ to $E_5(D_5)$, is shared by both series instead of being confined to a larger error in one series. Excessive contraction in this $A_3(G_3)$ to $E_5(D_5)$ region and also the excessive expansion around the $F_4(D\sharp_4)$ to $C_6(A\sharp_5)$ region is typical of all clarinets.

Figure 5(b) shows what extremely large errors occur for this same clarinet when tuned, by adjusting the barrel joint, to $B_4(A_4)$ at 440 cycles per second at a temperature of 71°F. Every tone of series I is too sharp, even though the errors in series II are as well balanced as possible.

Experiments on numerous clarinets show that unfortunately there is no definite room temperature recognized as normal and at which there should be optimum intonation. It should be noted that the temperature of the air within the tube is variable from room to breath temperature, the gradient being small from the middle of the tube to the mouthpiece.

The bore of the clarinet is essentially cylindrical and the B^b instrument has a diameter of nearly 1.50 cm. The flare for the bell joint begins at the $F_3(D\sharp_3)$ hole or slightly above. Variations from the cylindrical bore were found to improve the intonation of certain tones but only at the expense of others; and many trials showed this attempt at improvement to be impractical.

Figure 6, which shows that the standard bore gives certain inherent errors in the interval of a twelfth between the tones of series I and II, is a composite of results for many clarinets. It also shows a maximum contraction of about 0.25 semitone around $A_3(G_3)$; and a maximum expansion of 0.20 semitone around $C_6(A\sharp_5)$. The curve may be considered typical although some variation in the two points is common.

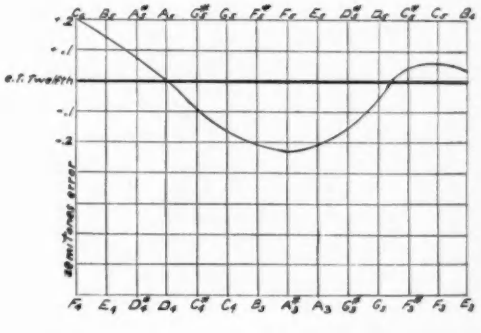

FIG. 6. Composite and typical errors in the equally tempered twelfth between related tones of series I and II. Plus errors indicate expanded intervals.

Assuming the typical errors for the twelfth intervals as in Fig. 6, it is still possible for the manufacturer, in placing the tone holes, to distribute the errors between series I and II in a favorable manner. Such a result is shown in Fig. 7.

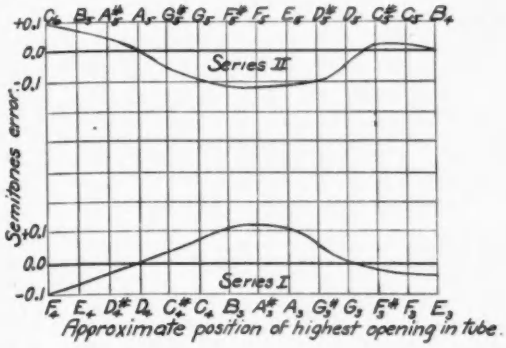

FIG. 7. Desirable distribution of errors in the twelfths between tones of series I and series II. However, the tuning tone $B_4(A_4)$ is correct and the entire error assumed by $E_5(D_5)$ of series I.

Such favorable design is found only in a few of the many clarinets tested and this advantage seems to be largely vitiated by neglect of any choice of a normal clarinet temperature. In fact, in most clarinets there seems to have been no attempt at a favorable distribution of errors between series I and II.

DISCUSSION OF RESULTS

It is obvious that to improve radically the clarinet intonation, errors as shown in Fig. 7 must be removed or reduced. To accomplish this, series I could be made independently correct at the time of manufacture by such hole placement as is indicated by the lower zero error line in Fig. 7. This would increase the errors of series II but as the register key is used only for series II, it could be employed to actuate mechanism which would add extra or auxiliary tone hole area to correct for excessive flatness around $E_5(D_5)$, and to subtract tone hole area for excessive sharpness around $C_6(A\sharp_5)$. The latter could be accomplished by lowering the position of the tampion with its pad which is involved in producing $C_6(A\sharp_5)$. Without these correcting mechanisms, the intonation would actually appear as in Fig. 6 where the tones of series I are pitched correctly and all errors are assumed by the tones of series II.

The experimental results corroborate the ideas of the late C. E. Potter, a piano tuner and clarinetist of Wilkes Barre, Pennsylvania, who actually built laboratory models of such a clarinet and relied upon his skill as a piano tuner to adjust their intonations. He added three extra auxiliary holes *4*, *5*, and *6* of Fig. 8, which are normally closed for sounding the tones of series I but which are opened, automatically and as required, by the action of register key *10* of Fig. 8 when the tones $E_5(D_5)$ to $A_5(G_5)$ of series II are being played. These require correction.

Also the register key mechanism reduces the tone hole area to correct for $B_5(A_5)$ and $C_6(A\sharp_5)$ of series II which are too sharp on almost all clarinets.

Fortunately these corrections, when made, also improve the higher tones $C\sharp_6(B_5)$ to $F\sharp_6(E_6)$ giving a superior intonation throughout the range of the clarinet. About 1932 such an instrument was given a practical and successful test by the solo clarinetist of the Philadelphia Orchestra, Mr. Robert E. McGinnis. This mechanism, which requires no new fingering and is not complicated, should certainly be adopted by manufacturers for the use of symphonic players or those who desire to play in tune without resorting to various annoying and impractical adjustments while performing.

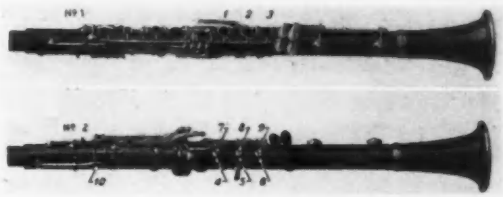

FIG. 8. Laboratory models of the Potter system. Holes *1*, *2*, and *3* are made slightly smaller than is now usual in order to flatten the tones usually too sharp in series I, i.e., $A_3(G_3)$ to $D_4(C_4)$. Auxiliary holes *4*, *5*, and *6* are opened only when the register key *10* is operated, increasing the effective areas of holes *1*, *2*, and *3*, respectively. This sharpens those tones of series II usually too flat, i.e., $E_5(D_5)$ to $A_5(G_5)$. These would be unbearably flat without auxiliary holes *4*, *5*, and *6*. Intonation is controlled by the manufacturer.

Air- and Bone-Conduction Audio Testing Assembly*

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(Received October 10, 1944)

ACCURATE, easily made tests of the hearing of the individual constitute much of the foundation for any adequate program for the prevention and correction of hearing impairment. The proposed air- and bone-conduction audio testing assembly obviates the use of a soundproofed booth or room if used in a moderately quiet room, eliminates many of the difficulties and errors of the usual methods of testing and test equipment, and provides the means for making many of the comprehensive tests previously available only in specialized acoustical laboratories.

In testing the hearing of an individual by means of the usual equipment and methods the following difficulties and errors are encountered:

1. When one attempts to obtain an air-conduction threshold for a normal ear, or one with small loss, in a so-called "quiet" room, that is, a room in which the noise level is 30–35 db, with an audiometer calibrated for use in a silent room, the room noise leaks under the earphone of the audiometer and masks the test tone, causing an apparent air-conduction hearing loss. Any attempt to compensate for this by shifting the zero of the audiometer is unsuccessful, because (a) the noise level in the test room is never the same in intensity and composition as that for which the attempted-compensation zero shift was made, and (b) a zero shift to compensate for noise leads to recorded hearing losses which are too small for ears which have fairly large true losses.

2. When one attempts to determine a bone-conduction threshold for a normal ear, or one with small loss, in the so-called "quiet" room, with an audiometer calibrated for a silent room, the room noise enters the ears by air conduction and masks the bone-conduction test tone, thus giving an apparent bone-conduction loss. The situation is worse in this case than in the determination of an air-conduction threshold, because for a bone-conduction threshold there is no air-conduction earphone to shut out the noise partially, and therefore it has its full masking effect. Any attempt to compensate by shifting the zero level has all the difficulties mentioned for the air-conduction case, augmented by the fact that, with the ears uncovered, differences in noise level and the resultant masking produce greater errors than in the case of air-conduction measurements.

The part of these difficulties arising from noises coming into the room from outside can be eliminated by testing in a soundproofed room, so quiet that even normal ears can hear no noise from outside. But the noises such as movement of feet, clicking of audiometer controls, etc., which arise within the room, may be present. Even when these are eliminated there will still be stray air radiation from the bone vibrator—radiation which at test frequencies above approximately 1800 cycles produces false, stray-air-radiation thresholds, rather than the bone-conduction thresholds which are desired.

Several "remedies" have been suggested to eliminate these difficulties. One such "remedy" has been the use of various kinds of ear plugs. Ear plugs which seal perfectly[1] cut out moderate room noise completely, but they also cause a shift of the bone-conduction threshold[2] by as much as 24 db at low frequencies, such as 250 cycles. Furthermore, ear plugs prevent any practical air-conduction measurements. Thus they are useless for this purpose.

A second proposed "remedy" has been the use of small enclosures, placed over the ears with acoustical seals where the enclosures touch the head. Such enclosures cut out room noise to a considerable extent, but they produce bone-conduction threshold shifts[3] of as much as 20 db

* Presented in the Symposium on the Correction and Prevention of Hearing Impairment, at the Twenty-Ninth Meeting of the Acoustical Society of America, in New York, May 12, 1944.

[1] N. A. Watson and V. O. Knudsen, J. Acous. Soc. Am. 15, 153–159 (1944).
[2] N. A. Watson and R. S. Gales, J. Acous. Soc. Am. 14 207–215 (1943), see especially Table I.
[3] See Reference 2, Tabe II.

at lower test frequencies. A *single* small enclosure does not keep out room noise because it covers only one ear, but it causes a bone-conduction threshold shift of as much as 15 db at lower test frequencies, i.e., up to 500 cycles per second. The air-conduction earphone with its sponge-rubber cap, frequently used to introduce masking noise into one ear while testing the opposite ear, approximates such a single enclosure and causes a bone-conduction threshold shift. Thus the masking introduced by the earphone is of little effect until a high level is reached—high enough to counteract the bone-conduction threshold shift. Just how much masking noise is wasted in this counteracting process is illustrated by the following example. The tests involved were made on a normal-hearing person in a silent room and thus were not influenced by room noise. For a bone-conduction test frequency of 250 cycles, with the vibrator applied at the center of the forehead, the mere presence of the masking earphone on one ear, with no masking sound emitted, gave a bone-conduction threshold shift of 17.5 db. As the masking sound ("thermal noise") was raised to a higher and higher level, there was no effect on the bone-conduction threshold until a thermal-noise level of 50 db above the thermal-noise threshold level for that ear was reached. Then the bone-conduction threshold began to return toward the normal open-canal bone-conduction threshold as the thermal-noise level was further increased. When the thermal noise reached a level of 72.5 db above its threshold, the bone-conduction threshold returned to normal. Thus 72.5 db of thermal noise was wasted, and for any level of thermal noise below 72.5 db above its threshold, the test tone was localized in the ear under the masking earphone, rather than in the ear to be tested. Further increase of the masking noise (above 72.5 db) shifted the localization of the test tone to the ear being tested, but gave a false threshold at a level higher than the correct bone-conduction threshold as determined with open ear canals in a silent room. Thus the thermal noise was useless for masking the 250-cycle tone for the first 72.5 db because of the bone-conduction-threshold shift; and it was useless at any values greater than 72.5 db because it, as well as the test tone, affected the ear being tested. Similar effects were found to occur at lower and higher frequencies. As much as 85 db of thermal noise (above threshold) was necessary to counteract the bone-conduction threshold shift at a frequency of 125 cycles per second.

The bone-threshold-shift effect was found *not* to occur if the size of the enclosures over the ears was increased enough—to approximately 2500 cc for the test range down to 125 cycles per second. (For the higher test frequencies smaller enclosures are sufficient.) Hence the larger enclosures provide protection from room noise without producing the bone-conduction-threshold shift.

On the basis of these results the air- and bone-conduction audio testing assembly was designed. It consists of an adjustable chair and chin rest; a pair of large adjustable, acoustically-treated air-conduction source boxes to be placed one over each ear; and an adjustable bone-conduction vibrator, mounted so as to impinge on the center of the forehead of the person being tested.

The performance of such a system is indicated by the results with the trial assembly at UCLA.

1. The air-conduction source boxes eliminate room noises with levels up to 40–45 db* for a normal hearing person. The exact level to which the noise can be raised without being heard depends on the type of test noise introduced by means of a loudspeaker into the soundproofed room[4] where the tests are conducted. Tests have been made with thermal noise, popular phonograph music, and pure tones. Thus the boxes exclude noises equivalent to the loudest ordinary noises measured by Currier[5] in some twelve rooms used by physicians for hearing tests.

2. The source boxes also shield against the stray air radiation from the bone vibrator so that accurate bone-conduction tests can be made at the high frequencies previously out of the range for accurate bone-conduction measurements. For example, at 4000 cycles (a frequency at which most bone vibrators, including the induction-type electrodynamic bone vibrator used in this assembly,[6] have strong air radiation)

* The noise was measured with a General Radio Type 759 Sound Level Meter, using the C, or "flat," network.

[4] N. A. Watson, J. Acous. Soc. Am. 9, 99 (1937), Fig. 1.

[5] W. D. Currier, Arch. Otolaryng. 38, 49–59 (1943).

[6] N. A. Watson, J. Acous. Soc. Am. 9, 99 (1937), Fig. 7.

a bone-conduction threshold is obtained by means of the assembly at a level 14 db above the stray-air-radiation threshold level, determined for open ear canals in a room silent except for the stray air radiation from the bone vibrator.

3. The large boxes do not give rise to false, enhanced bone-conduction thresholds, even at low frequencies—those for which enhancement by plugs or small enclosures is greatest.

4. The assembly allows air-conduction threshold measurements on either ear with the tone being applied by the air-conduction source on the back of the box corresponding to the ear tested, and the masking noise being applied from the air-conduction source on the back of the box on the opposite ear.

5. The assembly allows bone-conduction threshold measurements on either ear with the vibrator at the center of the forehead, air-conduction masking being applied to the opposite, not-to-be-tested ear by means of the air-conduction source at the outer end of the box.

6. The system is especially effective for testing cases in which one ear is normal and the other severely impaired. With usual methods (open ear canals and no masking) the bone vibrations always travel to the better ear no matter where the vibrator is applied to the head, and often the sound from the air-conduction earphone goes to the better ear, even when applied to the poorer ear. Besides these sounds from the test source, the normal, or nearly normal, ear is subjected to any room noise that may be present. The air-conduction source boxes of the assembly eliminate the effects of the noise on the better ear, yet do not cause artificial enhancement of the bone-conduction threshold as does a masking earphone of the ordinary type. With a masking sound (thermal noise) introduced into the source box on the better ear, an accurate bone-conduction-threshold measurement is possible in cases where it would not be possible using ordinary equipment and methods. For example, one observer had an 80-90-db air-conduction loss in one ear and practically none in the other. An accurate bone-conduction threshold taken for the poorer ear with the present assembly showed: (1) that the usual open-canal no-masking bone threshold was in error by as much as 50 db at some frequencies; (2) that the addition of an ordinary masking phone to the better ear (no masking sound) increased the error for the poorer ear by as much as 20 db at the lower frequencies, but that enclosure of the better ear in the air-conduction source box did not; (3) that the bone-conduction threshold for the poorer ear, determined with masking noise applied to the better ear by the usual masking phone, was in error by at least 10 db at the lower frequencies.

7. Equal loudness tests can be made for diagnosis and for use in specification of the best amplifier for compensating the individual's hearing loss[7] if adequate oscillators are available to feed alternately the air-conduction sources on the source boxes and the bone vibrator.

8. Articulation tests can be made readily if suitable microphone or phonograph, with associated amplifiers and attenuators, is available to feed the air-conduction sources and bone vibrator. These articulation tests can be made monaurally, diotically, or with various combinations of the air-conduction sources and the bone-conduction vibrator.

Thus the system provides, at relatively small expense, and without a soundproofed booth, the means for making, in a quiet but not silent test room, most of the extensive and intensive tests of hearing for diagnosis of hearing impairment and for specification of hearing amplifiers which previously could be made only in specialized acoustical laboratories.

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[7] N. A. Watson and V. O. Knudsen, J. Acous Soc. Am. **11**, 406–419 (1940).

Some Experimental Evidence for Peripheral Auditory Masking*

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(Received September 30, 1944)

THE microelectrode technique of recording auditory nerve potentials, as described by Galambos and Davis,[1] leads to characteristic oscillograms which are interpreted by them as caused by the activity of single nerve elements. The response to pure tones thus obtained is of great regularity and simplicity, and is elicited only by a frequency band characteristic for the nerve fibers under test.

The new technique, now in use in this laboratory and instituted with Dr. Galambos' kind personal assistance, is of great usefulness in the study of many auditory problems. One puzzling feature, already mentioned by the authors, is the comparative difficulty in obtaining a pure and satisfactory response. Although anesthesia and operative procedures, as well as preparation and insertion of the electrodes, are carefully kept as constant as possible, the results are quite unpredictable, even in animals with normal cochlear response. The interval between beginning of the anesthesia and time of recording seems to be of some influence, inasmuch as, in general, better results are achieved if a few hours are allowed to elapse between operation and testing. But one occasionally obtains animal preparations with apparently normal microphonics in which several hours of search in various places within, or on the surface of, the eighth nerve fail to yield the characteristic spike pattern for any pure frequency tested. Nevertheless, the nerve cannot be said to be inactive. Hissing sounds, as well as many other types of noise, usually lead to a loudspeaker response consisting of a fine crackling sound while the cathode-ray screen shows a complicated pattern of small spikes. It is possible that the difficulty just described is partly caused by a limitation in the equipment, since the oscillator used in this series of experiments does not generate frequencies above 15,000 c.p.s. But since, on the other hand, characteristic responses could be obtained for frequencies as low as 210 c.p.s., it seems puzzling that, in some experiments, one should not be able to contact a section of the nerve specific for *any* frequency below 15,000 c.p.s. The observation that noise leads to electric response comparatively easily might be taken as an indication that it is not so much the spectral composition of a sound that renders it effective as its sudden onset or, in other words, transient character. This assumption led to the use of the ticking of a wristwatch as test noise. The results seem to be of sufficient interest to be reported in the present paper.

Acoustically, the composition of the watch-tick is very complex, and an accurate analysis was not possible with the apparatus at our disposal. However, through the courtesy of Mr. B. Olney and his collaborators at the Stromberg Carlson Company, a simple qualitative analysis was obtained in the following way. The watch tick was picked up through a crystal microphone, amplified, and led into a loudspeaker after passing through an octave band-pass filter. It was evident that all octaves between 400 and 6400 c.p.s. contained a noticeable component. Higher octaves were not tested; but this partial analysis suffices to verify the complex character of the watch-tick.

TECHNIQUE

Ten cats, anesthetized with Dial (0.75 cc per kilo), were used. The bulla was approached by dorsal incision, opened, and inspected. The auditory nerve was exposed by opening the posterior cranial fossa and removing the cerebellum by means of a suction pipette. This method furnishes satisfactory visibility of the nerve and renders insertion of the microelectrode very easy. The pick-up technique was carried out

* Communication No. 62, prepared under the direction of Elmer A. Culler, from the Animal Hearing Institute, Laboratory of Psychology. This Institute is maintained by aid of the Research Council, American Otological Society. This paper was read at the twenty-ninth meeting of the Acoustical Society of America, May 12–13, 1944, New York, New York.

[1] R. Galambos and H. Davis, J. Neuro-Physiol. **6**, 39 (1943).

according to Galambos and Davis' original method. Amplification and oscillographic methods correspond to the common standard technique. The capacity-coupled amplifier has an input impedance of one megohm. Pictures were taken from the oscillograph by means of a constant speed camera. The wristwatch serving as

FIG. 1.

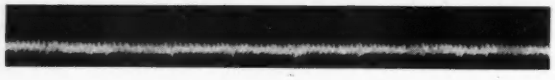


FIG. 2.

sound source was suspended close to the animal's ear, although in some sensitive preparations responses were obtained at a distance of over six feet.

As soon as the microelectrode touched or penetrated the surface of the nerve, some spontaneous activity and response to noise were usually observed through the monitor loudspeaker. In almost every case with normal cochlear microphonics, a distinct response to the watch-tick was obtained either immediately or after a slight shift of the microelectrode. This is in contrast with the fact that in some of the preparations no response to pure tones could be obtained for any frequency below 15,000 c.p.s. Through the loudspeaker the response sounds like a very good reproduction of the watch-tick, comparable to the response obtained from a crystal microphone. It is free from the sputtering noise characteristic for the nerve potential obtained by the microelectrode technique with pure tones.

The oscillogram (Fig. 1) shows a series of simple spikes or groups of a very few spikes of unequal size in which case one within each group is usually much taller than the others. The single spikes or groups of spikes are synchronous with the watch-tick. Because of heartbeat, respiratory and other occasional muscular activity, the base line is very irregular. This can be remedied by using an inductance in shunt with one of the coupling resistors (Fig. 2); in general, the latter type of arrangement was used throughout the experiment. It will be noticed that this change alters the spikes, giving them a more diphasic appearance, an indication that, in absence of a pure d.c. amplifier, great care must be taken in the interpretation of individual nerve spikes. The oscillograms of Figs. 1 and 2 were recorded with the microelectrode from a nerve fiber that showed no spontaneous activity and no response to pure tones of any frequency tested. In several ways they resemble the response to pure tone stimulation. In the first place, they, too, consist of spikes or small groups of spikes. Another important analogy is found in the fact that they can be suppressed by masking sounds. Galambos and Davis have shown[2,3] that pure tone response as recorded with the microelectrode can be masked by noise or even pure tones of appropriate frequency and intensity. Here, one is dealing with a response to a noise which can be suppressed by a pure tone. This is shown in Fig. 3. The masking

FIG. 3.

FIG. 4.

tone was generated by a beat frequency oscillator, amplifier, and loudspeaker, and conducted close to the animal's ear by means of a sound tube. The onset and presence of the tone were indicated by a glow lamp, fastened above the face of the oscillograph. A tone of 4050 c.p.s., about 45 db above human threshold (indicated by the bright line in Fig. 3), completely masks the electric response to the watch-tick. As in masking of pure tones, there is a frequency of maximal suppressing effect. In all experiments done with the same watch, this proved to be 4050 c.p.s. Figure 4 shows the influence of a tone of 3400

[2] R. Galambos and H. Davis, J. Acous Soc. Am. 15, 77 (1943).
[3] R. Galambos, J. Neuro-Physiol. (in print).

c.p.s. of equal intensity on the watch-tick response. No masking occurs.

In spite of these similarities between pure tone-and-tick response, there are fundamental differences. In the first place, there is evidence that the response is not caused by the activity of

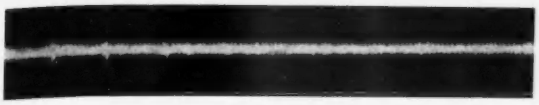


FIG. 5.

single nerve elements. The fact that there are often groups of spikes of different size would alone indicate that at least several fibers are involved. This becomes more obvious by the existence of "partial masking." Figure 5 shows the influence of a tone of 4600 c.p.s., again 45 db above human threshold. The tick response is partly abolished. It is hard to tell if this is caused by decrease in size of the spikes or to complete obliteration of one spike within the group. From some pictures the first assumption seems more probable. But, no matter how the result is interpreted, it indicates that the response is caused by more than one fiber. The type of response does not seem to depend entirely on the position of the microelectrode, since it can be obtained from various locations upon and within the auditory nerve. This latter result led to the obvious question, whether the tick response depends at all on the microelectrode technique and if one is justified in considering it as a pure nerve response. Experiments led to somewhat unexpected results. If the microelectrode is replaced by a simple, unshielded silver wire, the result is almost exactly the same. Even a silver wire loop applied to the round window yields practically identical oscillograms. Figures 1 to 5 were obtained from the same animal by the microelectrode technique. Figure 6 shows the watch-tick response as recorded from the round window of the same preparation. There is again complete masking by a tone of 4050 c.p.s. of 45 db above threshold. Figure 7 shows an instructive example of partial masking by a tone of 3800 c.p.s. of the same intensity (45 db), this time recorded from the round window. Figure 8 shows that a tone of 3600 c.p.s. (again of equal intensity) fails to cause masking.

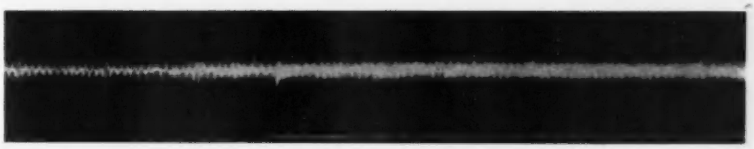
From the simple way in which these responses are obtained, one would feel inclined to think that the tick response would be, at least to a considerable part, microphonic in character. This would also explain why they can be picked up from various locations on the nerve. On the other hand, a predominantly microphonic origin of the tick response would seem to be unlikely in view of the possibility of practically complete masking by a rather narrow frequency band. To decide this question, use was made of the fact that nerve potentials disappear very soon after death, whereas microphonics undergo a characteristic, rather slow "death function." [4] This method has been used by Davis and his collaborators for the analysis of click responses recorded from the round window[5].

Three animal preparations were used for this experiment with identical results. As an example, consider first Fig. 9. It shows the watch-tick response as picked up from the round window. Complete masking is obtained by a tone of 4050 c.p.s. at an intensity of 55 db above threshold. The microphonic response to the masking tone is somewhat wider than the base line. With the electrode remaining *in situ*, the animal was killed by injection of 2 cc of Nembutal into the saphenous vein. Immediately after death of the animal ($2\frac{1}{2}$ minutes after the injection), the response to the watch-tick has practically ceased (Fig. 10). The microphonic response to the

[4] E. G. Wever, C. W. Bray, and M. Lawrence, Ann. Otol., etc. **50**, 317 (1941).

[5] S. S. Stevens and H. Davis, *Hearing* (John Wiley & Sons, Inc., New York, 1938), p. 328.

FIG. 6.



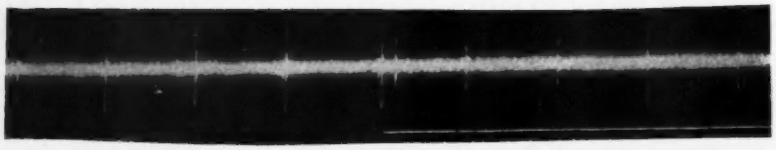


FIG. 7.

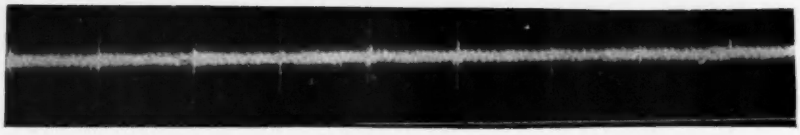


FIG. 8.

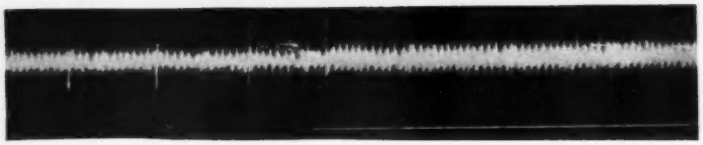


FIG. 9.

masking tone, however, is still visible as a broadening of the base line. Still two minutes later (Fig. 11), every nerve activity has stopped. There is still very strong microphonic response to a tone of 600 c.p.s. produced by the oscillator at the intensity setting used for the masking tone in Fig. 9. These experiments suggest that the response to watch-tick, even though recorded from the round window, is to a very great extent neural in origin, much unlike the round window potentials owing to louder clicks which seem to have a large microphonic component.[5]

Assuming now that the tick potential is generated by nervous tissue, two questions arise. First, which are the nerve elements involved, and second, what is the mechanism responsible for the masking of the tick response. Let us consider the second problem first. Purely acoustical interference seems very improbable and can easily be disproved experimentally. Figure 12 shows the watch-tick response as picked up from a crystal microphone. A 4050 cycle-tone, sufficiently strong to cancel the nerve response in an animal experiment, is picked up simultaneously, but no interference occurs. There is simple superposition.

Acoustical cancellation *within* the cochlea is very improbable in view of the difficulties encountered in trying to cancel even a pure tone response by another one.[6] Inhibitory action of the middle ear muscles would not be expected to lead to complete cancellation nor would it be easily understood why a rather narrow frequency band should be maximally effective. Furthermore, it is easily refuted by the nerve sectioning experiment (see below) which in one animal was performed with the opposite cochlea destroyed. In discussing inhibition of a single nerve fiber response by another tone, Galambos[3] comes to similar conclusions. Acoustical or mechanical interactions within the cochlea are discarded as possibilities. Neural interaction seems most probable, but the location cannot be estab-

[6] K. Lowy, J. Acous. Soc. Am. **14**, 2, 156 (1942).

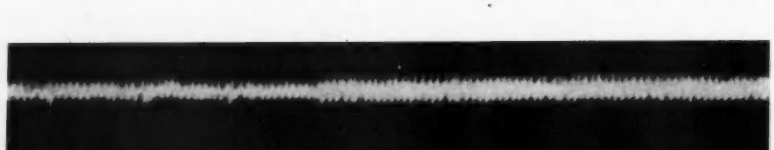


FIG. 10. 14.

FIG. 11.

FIG. 12.

FIG. 13.

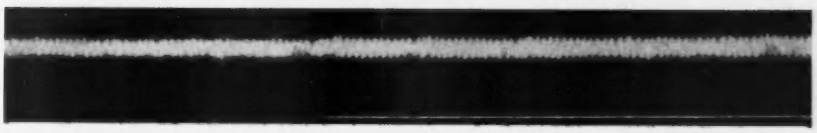
lished by direct experiment. The crucial test, namely, severing of the auditory nerve centrally from the site of the electrode, cannot be performed without breaking the microelectrode, or at least disturbing its position. Now, knowing that the watch-tick response, even if recorded from the round window, is neural in character, we are in a more favorable position. It is easy to record masking from the round window and then, without changing the position of the recording electrode, to interfere with the auditory nerve. This was done on three animals, always with the same result. Figure 13 was obtained from the same animal as Fig. 9, the only difference being that, two minutes before taking Fig. 13, the auditory nerve had been thoroughly infiltrated with novocaine. The two pictures are practically identical. Figure 14 was obtained after section of the auditory nerve near its entrance into the medulla. No difference is obtained. This seems to show that, no matter what its mechanism, the masking takes place in the peripheral organ without recourse to auditory centers. One would be inclined to accept these observations as objective proof for the existence of peripheral masking. To attempt any analysis of the particular mechanism involved would at present be pure speculation.

Also the question as to what structures are responsible for the watch-tick potential admits only of a conjectural answer. They must be located in the peripheral organ, since the response is not abolished by nerve section. Accepting the neural character of the potential and considering that it can be recorded equally well from the cochlea and from the auditory nerve, one could think of the spiral ganglion or its dendrites as a possible source. The large size of the potential might be caused by simultaneous stimulation of a great number of nervous elements.

It might be added that the optimal frequency for masking (4050 c.p.s. in our case) is not characteristic for a particular animal species. The same experiment was performed on two guinea pigs (electrode on tip of cochlea instead of round window). Again 4050 was the optimal masking frequency. Even in three human subjects with normal hearing, similar results were obtained. The subjects were placed in a convenient chair, the watch suspended nearby so that it could be easily heard. At the same time a tone of 4050 c.p.s. was generated by means of an oscillator and a loudspeaker, just loud enough to mask the watch-tick. Thereafter the frequency of the tone was slightly changed without alteration of the intensity setting. A very small shift in frequency in either direction was sufficient to make the watch-tick reappear. In one subject suffering from a moderate high tone deafness, however, the optimal masking frequency was found to be around 3000 c.p.s.

These observations tend to show that the frequency optimal for masking of the watch-tick is not dependent on the animal species tested. The results on animals suggest strongly that the subjective masking as observed in analogous experiments with humans, is of peripheral origin. Of

FIG. 14. 10.



course, they do not disprove the possible existence of central masking mechanisms.[7]

SUMMARY

1. The electric response of the cat's ear to a watch-tick consists of a characteristic rhythmic spike pattern.

2. This response can be recorded from the nerve with the microelectrode technique or by means of a plain wire, as well as from the round window.

3. The response can be masked by pure tones of suitable frequency and intensity within a limited frequency range. At the margin of this frequency range, "partial masking" is observed.

4. The watch-tick response ceases immediately after death. It is, therefore, considered of neural rather than of microphonic origin.

5. The process of masking must occur within the peripheral organ, since masking is not abolished by elimination of the auditory nerve.

6. Similar observations were made on guinea pigs; experiments involving subjective masking in human subjects gave analogous results.

[7] In fact, Kobrak, Lindsay, and Pearlman [Laryngscope **52**, 870 (1942)] report what they consider probable evidence against the peripheral origin of masking. However, it appears doubtful if their method is quite suitable for answering this question.

The Consideration of Hearing Aids and Audiometers by the Council on Physical Medicine*

HOWARD A. CARTER
American Medical Association, Chicago, Illinois

(Received September 5, 1944)

THE American Medical Association is composed of some 124,755 members. There are 2054 component county and district societies which go to make up 54 constituent associations in states and possessions.

The House of Delegates, which meets annually, is a representative body comprised of 175 members. The delegates are elected by the states and represent them according to population. Policies and regulations are formulated by this body. The House appoints the Board of Trustees and the officers of the association. One of the Councils which is an advisory body to the Board of Trustees is the Council on Physical Medicine.

The aims of the Council on Physical Medicine are: (1) to inform the medical profession and the public concerning the efficacy of apparatus recommended for medical purposes; (2) to protect the medical profession against fraud and misrepresentation associated with the manufacture and sale of such apparatus; (3) to advise the profession on the content of curricula for both undergraduate and postgraduate education in physical medicine; and (4) to promote research in physical medicine.

Although the practice of medicine is an art, the Council on Physical Medicine endeavors to evaluate apparatus and methods by strictly scientific procedures. The Council deals only with apparatus which is recommended and sold in connection with the practice of medicine. The Council on Pharmacy and Chemistry deals with drugs and the Council on Foods and Nutrition considers special purpose foods.

A glance through any surgical supply house catalog will reveal many pieces of apparatus that are used in the practice of medicine. It is practically impossible for the Council to investigate all the products, such as hospital furniture, beds, laundry machinery, wheel chairs, etc. Hence, the Council restricts its investigation to apparatus which has a definite therapeutic or diagnostic effect. The restriction goes further, that is, the physical agent must be applied directly to the patient.

It is expecting too much for a body of men the size of the Council to be completely familiar with all branches of medicine utilizing physical agents; hence, the Council has appointed groups of consultants to assist in its work.

The Consultants on Audiometers and Hearing Aids have been functioning since 1936. Many of the original Consultants are serving. They do this work gratis, the same as the Council members. Being advised by a group of Consultants on Audiometers and Hearing Aids, the Council has given consideration to audiometers and hearing aids and has published reports on them. Those hearing aids, as well as other therapeutic equipment that meet the rules of the Council are placed on the Council's accepted list and the manufacturer is permitted to use the seal of acceptance. A hearing aid or audiometer submitted to the Council is investigated according to the requirements for acceptance of these devices. The procedure is as follows: A product is submitted for consideration along with the required evidence. It is then referred to a specialist in the field who is asked to investigate the instrument and to review the evidence submitted by the promoters. If the Council's investigator can substantiate the evidence submitted by the firm, then the product is recommended for acceptance. The firm's report and the investigator's are referred to the Consultants, and if adopted by them, the Council is so advised.

One of the most perplexing problems is the lack of having any universally accepted method for testing hearing aids from a physical standpoint. It is hoped that this will be solved by the American Hearing Aid Association, an organization of manufacturers working with the American Standards Association. The Council will continue to test hearing aids with its own methods,

* Presented at meeting of the Acoustical Society of America May 12, 1944.

CHART 1.

124,755 Members (May, 1944)
2054 Component County and District Societies
54 Constituent Associations in States and Possessions
American Medical Association

House of Delegates
- Judicial Council
- Council on Scientific Assembly
- Council on Medical Education and Hospitals
- Council on Medical Service and Public Relations

Board of Trustees
- Journal A. M. A.
- Special Journals
- Library
- Committee on Scientific Exhibit
- Council on Pharmacy and Chemistry
- Chemical Laboratory
- Bureau of Investigation
- Bureau of Health Education
- Bureau of Legal Medicine and Legislation
- Council on Physical Medicine
- Council on Foods and Nutrition
- Bureau of Medical Economics
- Council on Industrial Health

that is, trying the instruments on patients. The proposed physical method will not test the value of bone conduction hearing aids and non-electrical instruments.

The Council is confronted with another problem which is very complex, namely, the evaluation of advertising matter which is used by firms for promoting physical medicine apparatus, including hearing aids and audiometers. One of the rules of the Council is that claims made for a device shall conform to the facts which are demonstrated. Misleading, unwarranted, and exaggerated statements are not permitted. If a firm persists in making them, its product is withdrawn from the list of accepted devices. Borderline cases are the most difficult to manage since opinions will differ.

Passing on the accuracy of the claims in advertising for a product frequently puts an undue strain on the Council's scientific methods. This difficulty is sometimes encountered when a "catchy" phrase is used to attract the public eye (or ear, as in radio plugs) to gain a commercial advantage over a competitor. This phrase is likely to be a spontaneous brain child of a highly paid but not too technical copywriter. As for example: "Deaf hear speech as clear as crystal," or "hear whispers with the X hearing aid" might be cited. These statements are misleading as viewed by the analytical mind of the pure scientist.

There is a fundamental difference in the philosophy of the scientific writer (engineer) and the advertising writer. The copywriter declares he must put his best foot forward—meaning he must startle the reader—or, in his opinion, he will not sell goods by the written word. An engineer with a scientific leading might prepare copy for a hearing aid as follows:

> In general it may be said, without any serious contradiction, the X hearing aid will assist hard-of-hearing individuals to hear the spoken sounds sufficiently well to carry on ordinary conversation, in a reasonably quiet place, at a distance of four feet between the speaker and the deafened person, provided the instru-

CHART 2.

The Council on Physical Medicine investigates the following apparatus:

- Anesthesia apparatus
- Anti-allergy apparatus
- Atomizers and assorted sprayers
- Audiometers
- Contraceptives
- Diathermy machines
- Fever therapy equipment
- Galvanic generators
- Hearing aids
- Heat applicators
- Infra-red generators
- Inhalators and resuscitators
- Metabolism, basal, test apparatus
- Oxygen therapy apparatus
- Passive vascular exercise apparatus
- Respirators
- Stethoscopes (electrical)
- Ultraviolet radiation equipment
- Vaporizers

By writing to Secretary, Council on Physical Medicine, A.M.A., 535 North Dearborn Street, Chicago, Illinois, a free pamphlet "Apparatus Accepted" may be obtained.

ment is fitted according to the most modern methods known, the batteries are not dead, the switch is turned on, and the wearer can read lips to supplement those sounds which are not heard clearly with the receiver.

The advertising writer would probably comment most violently in undignified language that precious and expensive advertising space is being wasted merely to say nothing, and "if the firm did not have me, a promotional expert, around here to advise, the company would go bankrupt." The advertising writer would promptly put his best foot forward and revise the copy as follows:

> Ladies! Do you want to miss what your friends are saying about you at the Elm Street Circle? Only the X hearing aid assures you of normal, effortless hearing; batteries last twice as long.

The foregoing examples are exaggerated and to the writer's knowledge they do not exist, but they illustrate the point.

Not many of the manufacturers of hearing aids goebbelize their advertising. When violations of the rules appear in print, the firm is informed, given a chance to revise or offer an explanation, and if the rules are not met, after due consideration the firm is informed that withdrawal of acceptance will proceed. This is the only means of policing the Council has and is different from that of the F.T.C., which has greater power. The purpose of the Council is to advise the profession and it is supported by the physicians for this purpose. The Council members and the consultants give their services gratuitously.

Insofar as it is known, a great majority of manufacturers have confidence in the objective attitude and integrity of the Council on Physical Medicine and in its Consultants on Audiometers and Hearing Aids. Lastly, it is advisable to maintain pleasant relations among members of the profession, manufacturers, and the Council, and so abide faith, hope, and a sense of humor. The greatest of these is a sense of humor.

CHART 3.

The Council on Physical Medicine has the following standing groups of consultants:

Consultants on:

- Audiometers and hearing aids
- Artificial limbs
- Electrocardiographs
- Education
- Ophthalmic devices
- Respirators
- Roentgen rays

The Production of Inharmonic Subfrequencies by a Loudspeaker

R. V. L. HARTLEY
Bell Telephone Laboratories, Incorporated, Murray Hill, New Jersey
(Received September 19, 1944)

A RECENT paper[1] by Olson discusses those peculiarities in the behavior of a loudspeaker which arise from the non-linearity of its elastic support. These include the production of subharmonics of the driving frequency. The discussion is based on an idealized system which has only one resonant frequency, in the case when the displacement is so small that the non-linear effects are negligible. An actual system, of course, has more than one resonant frequency.

The purpose of this note is to point out that the theory[2] of multiresonant non-linear systems indicates that inharmonic subfrequencies may also be produced. For example, if p_1 and p_2 are two such resonant frequencies, sustained oscillations at those frequencies can be maintained at the expense of energy supplied by a driving force of frequency p_0, provided the frequencies satisfy the relation

$$2p_1+p_2=p_0. \tag{1}$$

This is for the type of non-linearity discussed by Olson, where

$$f=\alpha x+\beta x^3.$$

If the force includes a term proportional to x^2, then oscillations are possible which satisfy the relation

$$p_1+p_2=p_0.$$

(1) is satisfied if

$$p_1=p_2=p_0/3.$$

This represents the production of the third subharmonic and requires only a single resonant frequency. Thus the subharmonics mentioned in the paper may be regarded as a special case of a more general class of subfrequencies.

[1] Harry F. Olson, J. Acous. Soc. Am. **16**, 1 (1944).
[2] R. L. V. Hartley, Bell Sys. Tech. J. **15**, 424 (1936).

Letter to the Editor

The Editorial Board assumes no responsibility for the opinions expressed in letters.

Answer to Schier's Letter to the Editor

FREDERICK M. GROSSMAN AND CHARLES T. MOLLOY
New York, New York
November 12, 1944

WE regret to find it necessary to submit the following reply to the Letter to the Editor[1] by Mayer B. A. Schier appearing in the October, 1944 issue of *The Journal of the Acoustical Society of America*. The contents of Schier's letter leave us with the definite impression that he has overlooked the purpose of, and the evidence presented in, our paper[2] and that he has included a large amount of irrelevant matter.

Schier refers to our paper[2] in his first and second paragraphs only. The rest of his letter refers not to our paper but either attacks previous publications of one of the authors (Grossman) or presents Schier's own views on the fitting of ear molds.

In his first paragraph, Schier makes a general derogatory statement under the terms "corrective comments" and "in the interest of accuracy." We take exception to the use of these terms and submit that Schier has demonstrated neither the need for correction, nor the existence of inaccuracies in our paper.

Schier stated in his second paragraph: "This paper endeavors by laboratory demonstration to substantiate certain theories which Grossman offers as a basis of a supposedly new or novel type of individual earpiece for the conduction of the higher frequencies emitted from the receiver of a hearing aid into the deafened ear of nerve type cases." In our paper,[2] however, our purpose was explicitly stated as: "to study experimentally, variations of the acoustic line between the receiver of a hearing aid and the drum, and to supply a mathematical analysis for the phenomena involved." Our investigation was initiated by an opinion of Littler,[3] by the results of clinical experiments reported by Halsted and Grossman,[4] and by further experiments of Grossman's.[5] Apart from his general adverse remarks, our paper received no critical treatment by Schier, and therefore, there can be no discussion of his reference to our paper.

The following paragraphs are answers to assertions irrelevant to our paper but they are made necessary because of the publication of these assertions.

In his letter,[1] Schier states: "In practical form he (Grossman) suggests an earpiece with a very short tip, a sound bore of extreme diameter and interposed between earpiece and receiver a little gadget presumably to vary the air column. It is a matter of fact that the intent and practice are neither new nor novel." Halsted and Grossman's[4] recommendation, based on clinical experience, reads: "If it is decided to connect the receiver with the individual ear mold, the length of the tip and the size of the sound conveying canal can be prescribed. In cases with perceptive hearing impairment an acoustic high pass filter may be used with advantage." This statement, i.e., the description of the utilization of changes in the acoustic line between hearing aid receiver and drum, is the first, appearing in the literature on the subject, accessible to us. What Schier calls the "little gadget" in his letter, we gather (but cannot be sure, of course) refers to the acoustic filter used in the line between receiver and drum. It should be noted, in this connection, that Grossman holds the U.S. Patent No. 2,363,175 for this filter.

It is an unwritten rule, in scientific communications everywhere, that specific references support an author's assertions. Schier, however, has given no reference whatever to support his accusations. In view of the above, Halsted and Grossman take exception to Schier's unsupported statement: "It is a matter of fact that the intent and the practice are neither new nor novel."

As to the "focal point of acoustic engineers, physicists, and governmental agencies," [1] namely, "the practice of having every air conduction instrument fitted with an individual earpiece," [1] Schier claims to have "initiated the practice." [1] We regret that we are obliged to disagree with Schier's priority claim. Itard[6] reported, in 1842, that the invention of individual earpieces, for the purpose of fastening air conduction instruments to the ear, was submitted by Gateau and Eon to the French Academy of Medicine. It was favorably reported by Mr. Thillaye. Thereupon the Academy recommended its use.

In the fourth paragraph of Schier's letter[1] he accuses Grossman of inaccurate reference to Schier's work. In the one paper by Schier on the subject of earpieces,[7] apart from his letter,[1] his only positive recommendation with regard to the length of the tip of the earpiece reads as follows: "That portion known as the actual tip should be as long as comfortable depth of entry will permit." Halsted and Grossman,[4] in discussing middle ear impedance hearing impairment, state: "The ear mold should have a long tip (here the reference to Schier's paper is given) and a sound converying canal of approximately 3 mm." Grossman's[5] only other reference to Schier reads: "It is an old practice in fitting a patient who has middle ear hearing impairment with a hearing aid to make the tip of the ear mold as long as possible. The design has been advocated lately and the principle of the use generalized by Schier." These are the quotations which Schier chooses to call: "a very inaccurate explanation of and reference to my advocacies."

Since the authors are confining themselves to answering Schier's accusations they offer no comment on his own "personal concepts" set forth in paragraphs 5 to 9 of his letter.

In paragraph 10 he describes a "little experiment" with a figure which was projected on the screen after the presentation of our paper before the Acoustical Society of America at the New York Meeting in 1944. We welcome

this occasion to have our comments recorded in the Journal since we were unable to answer Schier at the meeting for want of time.

This is how Schier describes the experiment he refers to: "An instrument is placed on an electric ear and its characteristics graphed. I made three little tube fittings in simulation of earpiece sound channels. With all conditions fixed as in the graphing of the instrument alone, with the first tube there was . . ." (Here the result of the experiment is given.) Schier then proceeds: "The experimental demonstration of clinical experiences was performed for me by qualified acoustical engineers and a physicist (no names are given) in April, 1939."

We wish to draw attention to the fact that Schier's experiment and figure appear for the first time in his letter of October, 1944.[1] Both the description of the experiment and the figure are conspicuously absent from, and are not even mentioned in, the only paper on earpieces[7] ever published by Schier (to our knowledge); namely, his article of January, 1941,[7] which was accepted for publication a year earlier, on January 15, 1940. In it no mention is made of the "Schier tubes" which are "all of the same diameter, excepting that filtration and diversion efforts were made to change the characteristics of sound. . . ."[1] The reader will recognize that the experiment and figure of Schier would not have served as a basis for serious consideration in clinical, experimental, or theoretical work even had they been published previous to the publication data of papers mentioned in references 2, 4, and 5.

We feel that the discussion of the above issues can be considered reasonably complete. All material bearing on these issues has appeared in print and the necessary references have been given. We especially recommend in this connection the publication of Schier.[7]

[1] M. B. A. Schier, J. Acous. Soc. Am. **16**, 126 (1944).
[2] F. M. Grossman and C. T. Molloy, J. Acous. Soc. Am. **16**, 52 (1944).
[3] T. S. Littler, J. Sci. Inst. **13**, 144 (1936).
[4] T. H. Halsted and F. M. Grossman, N. Y. State J. Med. **42**, 1944 (1942).
[5] F. M. Grossman, Arch. Otolaryng. **38**, 101 (1943).
[6] J. M. G. Itard, *Traité des Maladies de l'Oreille et de l'Audition* (Mequinon Marvis Fils, Paris, 1842), second edition, Vol. 1, pp. 443, 444.
[7] M. B. A. Schier, Laryngoscope **51**, 52 (1941).

November 9, 1944

Dear Professor Firestone:

I am in full agreement with the answer of Grossman and Molloy to the Letter to the Editor by Mr. M. B. A. Schier, published in the October, 1944 issue of the Journal.

Very truly yours,

THOMAS H. HALSTED, M.D.

Acoustical Society News

Spring Meeting, 1945

The next meeting of the Acoustical Society will be held in the Hotel Pennsylvania, New York City on Friday and Saturday, May 11 and 12, 1945. Dr. Leo Beranek is Chairman of the Program Committee and those wishing to submit papers for presentation at this meeting should mail their titles and abstracts to him at Cruft Laboratory, Harvard University, Cambridge, Massachusetts. Inasmuch as the registered attendance at the meeting last spring was 340, it is anticipated that the forthcoming meeting will be well attended and that a significant program will be assembled.

New Members

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Jamaica Plain 30
(Boston) Massachusetts
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Underwater Sound Laboratory
Harvard University
Cambridge 38, Massachusetts

Chauncey D. Bond
Weaver Piano Company, Inc.
Broad and Walnut
York, Pennsylvania
Vice President and General Manager

Gilbert G. Brown
Newark Athletic Club
Newark, New Jersey
Sales Engineer
Federal Telephone and Radio Corporation
591 Broad Street
Newark, New Jersey

Mark W. Bullock
1609 South 24 Street
Lincoln 2, Nebraska
Chief Engineer Radio Stations KFAB, KOIL, KFOR
KFAB Broadcasting Company
9 and P Streets
Lincoln 1, Nebraska

Donald L. Clark
298 Hazelwood Terrace
Rochester 9, New York
Assistant Engineer
Stromberg Carlson Company
Rochester, New York

Philbrook Cushing
301 Mt. Kemble Avenue
Morristown, New Jersey
Radio Engineer
Aircraft Radio Corporation
Morristown, New Jersey

Harry J. Donaldson
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Chicago, Illinois
Operating Superintendent
Motiograph
120 North Green Street
Chicago, Illinois

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Hoboken, New Jersey
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Sperry Products, Inc.

Harold L. Ericson
10 Mayflower Road
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Special Research Associate
Cruft Laboratory
Harvard University
Cambridge 38, Massachusetts

George J. V. Faley
219 Ridge Road
Rutherford, New Jersey
Research Engineer
Bell Telephone Laboratories
Murray Hill, New Jersey

Ulrich R. Furst
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Fellows	179
Members	832
Total Membership	1011

Current Publications on Acoustics

F. A. FIRESTONE
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Book Reviews

The Technique of Motion Picture Production. A symposium of eleven papers presented at the semi-annual convention of the Society of Motion Picture Engineers, Hollywood, California. Published for the Society of Motion Picture Engineers by Interscience Publishers, Inc., New York, 1944.

The evident purpose of the symposium and of the book is to give the reader a comprehensive picture of the manner in which the various parts of the work of motion picture production are carried on. Much emphasis is laid on the revolutionary changes which came with the advent of sound, on the increased application of scientific methods of control, and on the high degree of specialization required in each of the many functions represented. To your reviewer, the most impressive thoughts derived from reading the book are the complexity of the operations, and still more the complexity of the organizations, and the remarkable job of coordination which must be accomplished in integrating the work of the many departments and specialists into an artistic and effective whole, while still giving the specialized experts latitude to make artistic as well as technical contributions. In days of simpler operations, the director must largely have conceived the form that the final presentation was to take, specifying the scenery, just what pictures were to be taken, and how presented, and where music was to be introduced, and what kind. Today the director has the final say, but the set designers, costume designers, and color experts apply their skills, the camera man works for the desired photographic effects, and takes numerous alternative shots, the lighting experts anticipate the requirements of the cameraman, the musical director decides what music is wanted, the re-recording "mixer" uses his judgment about sound quality and sound effects, laboratory technicians decide how pictures are to be printed, and the editor selects and arranges the scenes for best telling the story and dramatic effects.

Through many of the papers audience psychology is an uppermost theme, and many points are illustrated by citations from well-known pictures. It must not be inferred that organization is a major theme. Nearly all of the papers are devoted to the problems and operating methods of the department or field of work that the author is discussing. They were presented for readers interested in the business rather than as popular expositions, and some of the descriptions are not easy for an uninitiated reader to follow, particularly since the motion picture industry has evolved a terminology that is scarcely used outside. Despite such difficulties as technical terms, and the fact that many operations and techniques are so complicated that they are not easily explained in brief space, the reader will find the book exceedingly informative. Those who are interested in the fascinating subject of photographic tricks will find many examples of how these are used, but with a few exceptions, without enough detail for a very clear understanding.

Members of the Acoustical Society will scarcely find any difficulty following the discussions in the three chapters that deal primarily with sound, namely, the one on "Sound recording," by Homer G. Tasker, which goes into fundamentals, and explains the special acoustical problems presented by motion picture conditions; the one on "Re-recording," by Lloyd T. Goldsmith; and the one on "Prescoring, and scoring," by Bernard B. Brown.

The following chapter headings will give an idea of the topics discussed. I. Technology in the art of producing motion pictures; II. Cinematography in the Hollywood studios: Black and white cinematography, putting clouds into exterior scenes, technicolor cinematography; III. Special photographic effects; IV. Re-recording sound motion pictures; V. The technique of production sound recording; VI. Prescoring and scoring; VII. Illumination in motion picture production; VIII. The Paramount transparency process projection equipment; IX. Motion picture laboratory practices; X. The cutting and editing of motion pictures; XI. The projection of motion pictures.

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RCA Victor Division
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Speech Reading—Jena Method. ANNA M. BUNGER. Pp. 136. The Interstate Printers and Publishers, Danville, Illinois, 1944. Price $2.50.

As stated by the editor in his foreword, this is the only American textbook which presents the Jena method of speech reading. This method has three definite aims: (1) the development of speech consciousness, (2) the imitation of speech movements with (3) all possible help from rhythm. By these means there are built up rapid, almost automatic recognition and interpretation of rhythmic syllables. Later these same syllables, so well established, are converted into rhythmic phrases of everyday conversation.

Following a discussion of the three aims there are lesson plans illustrating methods for the acquisition of these techniques of awareness of movements and of imitation of the speaker. There are drills on syllables in various rhythms followed by practice material in words, phrases, and sentences based upon real life experiences. Reading by clues is stressed, that is, the skill of deriving meaning quickly and easily from the context in contrast to a laborious attempt to speech read word by word.

Actual conversation is stimulated and developed by informal talks, stories of real happenings, and question and answer games. As in the case of the foregoing drills and exercises, these are based upon phrases and sentences likely to be in daily use by the student of speech reading and his associates.

Adaptations of the Jena method and of the lesson plans are suggested for use with hard-of-hearing and deaf children. At the end of the book is a list of words most commonly used.

A great value of this text lies in the fact that the excellent practice materials can be used by all teachers of speech reading whether or not they are exponents of the Jena method. Built upon real life needs and situations, the lesson plans provide ready-made exercises for hours of practice, plus suggestions and possibilities for the enrichment and planning of many more. The author has made a splendid contribution to the literature for those who teach lip reading to hard-of-hearing adults and to deaf and hard-of-hearing children.

HARRIET M. DUNN
University of Michigan

THE JOURNAL OF THE ACOUSTICAL SOCIETY OF AMERICA VOLUME 16, NUMBER 3 JANUARY, 1945

References to Contemporary Papers on Acoustics

MOST of the titles of papers appearing in other languages than English have been translated. Abstracts in English of many foreign papers have appeared or soon will appear in Science Abstracts, Section A. Where references are made to Science Abstracts, the reference is to the volume number and abstract number. The abbreviations of the names of journals are those used in Science Abstracts and can be found in any annual index to those abstracts.

The numbers appearing at the head of each entry are the numbers which identify the subject headings in the Cumulative Index of this journal issued in November, 1939. The same Classification of Subjects can be found in the index to Volume 15.

Compiled with the generous assistance of Samuel Sass, Physics Librarian, University of Michigan.

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2.11 **If You Can't Kill It, Isolate It.** (Isolating Machinery which is a Source of Noise and Vibration by Means of Elastic Suspensions.) H. H. FINK. Scientific American **171**, 118–120 (Sept. 1944).

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4.1 **Further Studies on the Physiological Mechanism of Acoustic After-Illusion Relating to Rhythm Frequency in Man.** I. I. KOROTKIN. Fiziol. Zh. U.S.S.R. **28**, 411–420 (1940).

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10.9 **Catching up with Nature: Blind Flying.** C. F. KETTERING. Vital Speeches **10**, 736 (Sept. 15, 1944).

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THE JOURNAL OF THE ACOUSTICAL SOCIETY OF AMERICA VOLUME 16, NUMBER 3 JANUARY, 1945

Review of Acoustical Patents

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Printed copies of patents are furnished by the Patent Office at a cost of 10¢ each. Address all communications to: The Commissioner of Patents, Washington, D. C. Remittance may be made by money order, certified check, cash, or special coupons which are sold in blocks of twenty or books of one hundred.

The decimal numbers appearing before titles are those which identify the headings being used in the analytic subject index of this journal. The Patent Office classification given in parentheses with each patent entry is outlined in the "Manual of Classification of Patents" which is obtainable from the Superintendent of Documents for $1.50.

Unless otherwise indicated, patents listed have been issued on the date shown by the United States Patent Office.

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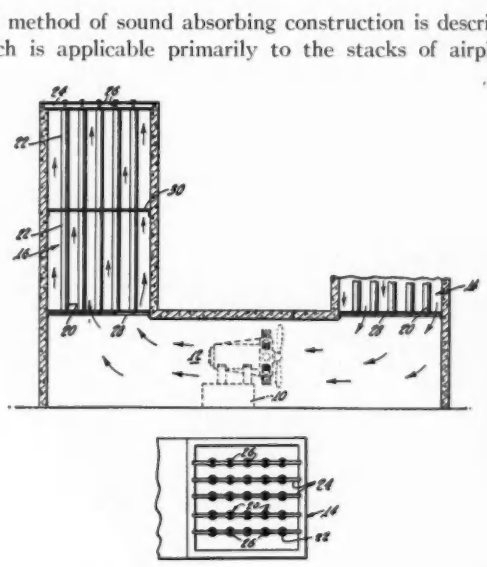
2,334,502

2.1 SOUND ABSORBING STRUCTURE

John S. Parkinson and William A. Jack, 3rd, assignors to Johns-Manville Corporation.
November 16, 1943, 19 Claims (Cl. 181–35).

A method of sound absorbing construction is described which is applicable primarily to the stacks of airplane



engine test cells. The sound absorbing elements consist of cylindrical columns having an outer surface of perforated

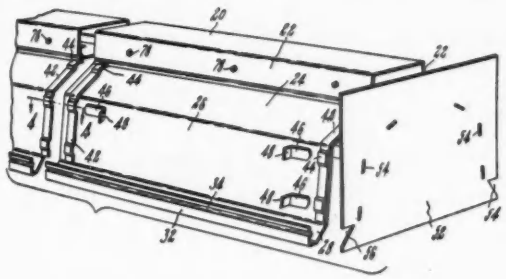
metal or asbestos board and a filler of loose or solid absorbent element. One modification covers the use of a solid or hollow unit of cast porous concrete. It is claimed that for a given cross-sectional area of treatment, this construction is on the order of 60 to 70 percent more efficient than "ribbon wall" methods of construction.—HJS

2,338,521

2.1 HOUSING FOR FLUORESCENT LAMPS

Barnett Levy, assignor to A. L. Smith Iron Company.
January 4, 1944, 4 Claims (Cl. 240–78).

A fluorescent lighting trough adaptable to installation in a perforated metal pan type of acoustical ceiling. The



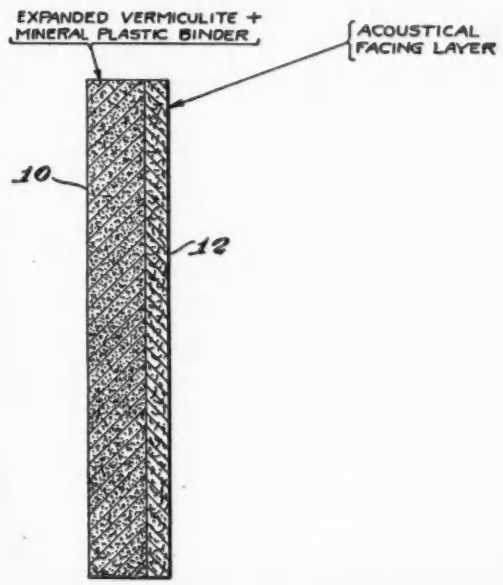
patented feature is the means of coupling successive lengths of trough and the attachment of end plates.—HJS

2,340,535

2.4 BUILDING MATERIAL

Paul W. Jenkins, assignor to H. H. Robertson Company.
February 1, 1944, 2 Claims (Cl. 72–26).

This patent describes a fireproofing tile or slab, for protecting the under side of a metal floor, combined with an



acoustical material. The fireproofing slab is composed of vermiculite with gypsum binder, and the acoustical material is any of a number of commercial incombustible products. It is bonded to the slab during manufacture of the composite unit.—HJS

2,338,813

2.4 METHOD OF MANUFACTURING SOUND DEADENING MATERIAL

Otto W. Hueter, assignor to Mitchell and Smith, Incorporated.
January 11, 1944, 2 Claims (Cl. 92–55).

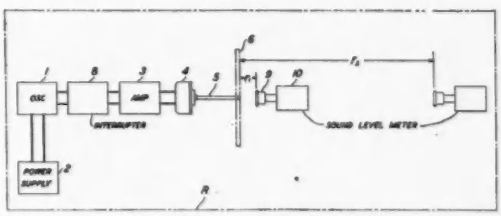
This patent describes the manufacture of an acoustical tile formed of rock wool with a starch binder. The rock wool is in the form of "bolls" which are saturated with the binder, pressed into a slab, and baked. The resulting crust is sawed off of the face side thus exposing voids and fissures having the appearance of porous travertine. The crust is left on the back side as breathing stop.—HJS

2,356,478

2.8 METHOD OF ACOUSTIC MEASUREMENT

Norman R. Stryker, assignor to Bell Telephone Laboratories, Incorporated.
August 22, 1944, 6 Claims (Cl. 181–0.5).

In determining the sound absorption in a room (as by the reverberation method) caution must be observed to minimize the effect of standing waves. It is said that the use of short pulses of sound (of the order of 3 milliseconds) accomplishes this same result. The pulse of sound is radiated from a tube **5** of small diameter such that the end



(which may be equipped with a baffle **6**) acts as a point source. The energy density is measured at two positions in the room distant r_1 and r_2 from the source. (Distances used in a room of 10,000 cubic feet in volume were one and 16 feet, respectively.) Based on the formulae for continuous sound, when no baffle is used the total absorption A is calculated by

$$A = \frac{16\pi(1-R)r_1^2r_2^2S}{S(Rr_1^2-r_2^2)+16\pi(1-R)r_1^2r_2^2},$$

where R is the ratio of the sound energy densities obtained from the antilogarithm of the difference of the two sound level meter readings; S is the total area of the room.

Although the oscillator **1** is set for discrete predetermined frequencies a short pulse is used, so in effect the

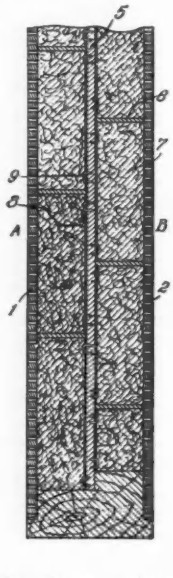
room is insonified with a continuous spectrum whose principal frequency band is inversely proportional to the length of the pulse. For the pulse length suggested the method thus gives an absorption averaged over a frequency band approaching 400 cycles/sec. wide.—RWY

2,350,513

2.9 SOUND BARRIER WALL OR DOOR CONSTRUCTION

Ralph L. Leadbetter, assignor to Burgess-Manning Company.
June 6, 1944, 9 Claims (Cl. 20–91).

A novel type of sound insulating construction is described. The outer surfaces of perforated board are backed by sound absorbing material. The air-impervious diaphragm **5** in the center may be damped either by externally applied layers or by being composed of non-vibratile material, and may be stiffened by the attachment of grids



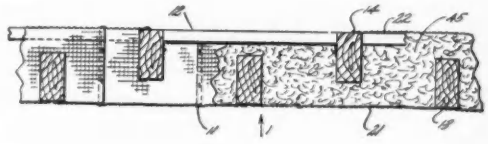
6. Attenuation is thought to take place chiefly on the receiving side **B** of the center diaphragm through the action of the absorbing material in "decoupling" the diaphragm from the air. It is believed that sound pressure is only negligibly reduced between the outer surface on the source side **A** and the diaphragm, the chief function of the absorption on the source side being to reduce the sound level in the source room.—HJS

2,341,305

2.12 SOUND DEADENING WALL CONSTRUCTION

Carroll L. Woodworth, assignor to Gladding, McBean & Company.
February 8, 1944, 3 Claims (Cl. 72–16).

The presence of a solid wood fire stop bridged across the air space in a double wood stud partition tends to nullify the sound insulating efficiency gained by the double construction. This patent describes a fire stop formed of an



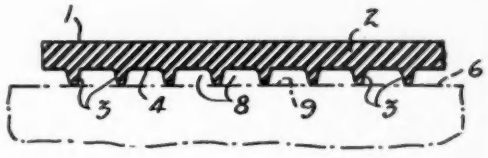
elongated wire mesh basket packed with a fireproof fill such as vermiculite. The construction restricts updraft effectively but transmits negligible vibration from one side of the wall to the other.—HJS

2,337,525

2.12 SOUND DEADENER

Paul G. Peik, assignor to The General Tire & Rubber Company.
December 21, 1943, 13 Claims (Cl. 20–6).

A rubber mat for deadening the vibration of metal floor panels in automobiles. Superior efficiency is obtained by means of molded ribs which contact the floor and absorb



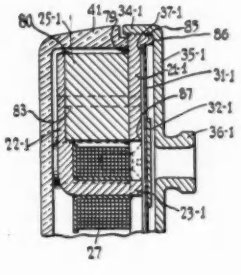
vibratory energy through a wiping action. The patent also covers the cutting of slits or elongated holes through the mat, which further increase its effectiveness.—HJS

2,344,023

4.5 EARPHONE

Richard W. Carlisle, assignor to Sonotone Corporation.
March 14, 1944, 6 Claims (Cl. 179–107).

The earphone described in this patent is an insert type hearing aid. The patent, for the most part, is devoted to



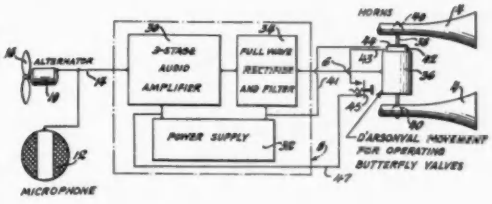
detailed description of the mechanical construction, which allows the receiver size to be reduced.—FHS

2,350,390

5.1 HORN CONTROL SYSTEM

Robert M. Critchfield, Paul L. Schneider, and Brooks H. Short, assignors to General Motors Corporation.
June 6, 1944, 4 Claims (Cl. 177–7).

An automobile horn which is too loud for a slowly moving car in a residential district may not be loud enough on a rapidly moving car in a noisy manufacturing district. An object of this invention is to provide a horn whose



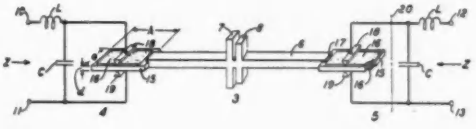
loudness increases with car speed and/or ambient noise. The motion of the air past the propeller **16** results in a voltage from alternator **10** which opens the butterfly valves **40** as the car speed increases. Ambient noise picked up by microphone **12** likewise can cause the butterfly valves to open. When the horn button is depressed switch **45** energizes a brake which prevents the valves **40** from changing their setting while the horns are sounding.—RWY

2,345,491

5.4 WAVE TRANSMISSION NETWORK

Warren P. Mason, assignor to Bell Telephone Laboratories, Incorporated.
March 28, 1944, 34 Claims (Cl. 178–44).

This patent teaches how electromechanical wave filters, utilizing longitudinal vibrations of acoustic rods together with flexural vibration of cross bars to provide attenuation peaks at the edges of the pass band, may be constructed.



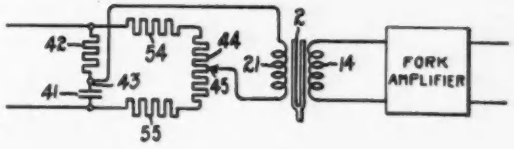
Piezoelectric crystals **15** such as Rochelle salt are used as the electromechanical converters in conjunction with the mechanical filters.—LWS

2,346,984

5.7 RATE CONTROL FOR TUNING FORK OSCILLATORS

Milton S. Mead, Jr., assignor to General Electric Company.
April 18, 1944, 7 Claims (Cl. 250–36).

This invention relates to the method of controlling the frequency of a tuning fork oscillator by adjusting the phase between the driving force and vibrations of the fork. This is accomplished by inserting a phase-shifting bridge in



either the driving or pick-up circuit of the oscillator. The phase-shift adjustment may be calibrated in terms of change in fork rate, such as seconds per day.—LWS

2,351,690

5.8 ELECTRIC HORN

Oliver Lucas, assignor to Joseph Lucas Limited.
June 20, 1944, 3 Claims (Cl. 177–7).

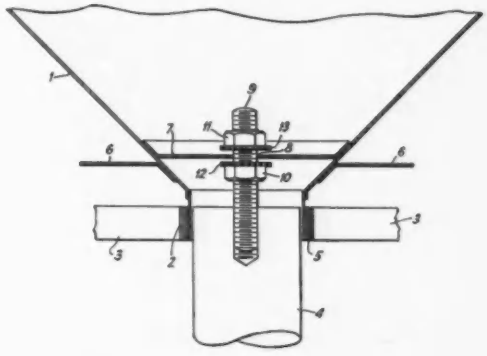
This is a two-tone auto horn.—FHS

2,347,799

5.8 LOUDSPEAKER

Alexander Schaaf; vested in the Alien Property Custodian.
May 2, 1944, 3 Claims (Cl. 179–115.5).

Two stops, **10** and **11**, limit the motion of the loudspeaker



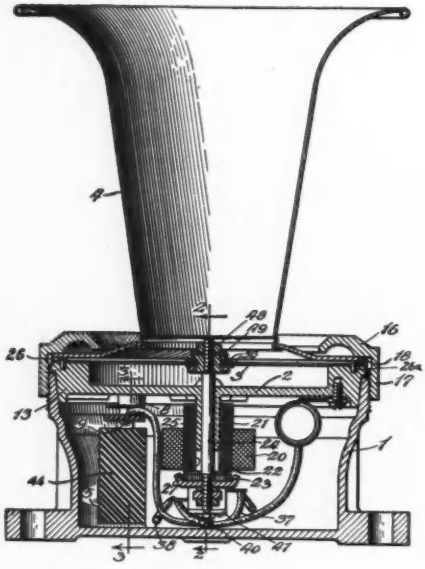
cone so as to prevent damage by exceptionally large signals.—FHS

2,352,602

5.8 ELECTRIC HORN

E. H. Church, assignor to Benjamin Electric Manufacturing Company.
July 4, 1944, 6 Claims (Cl. 177–7).

The housing which encloses the electrical components of this horn is designed to prevent the escape of a flame from



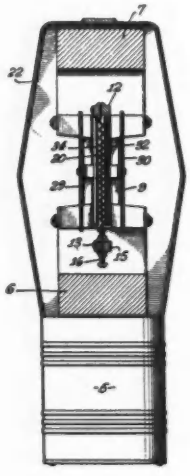
within the housing, and thus prevent the ignition of any inflammable mixture in the vicinity of the horn.—FHS

2,346,395

5.9 SOUND PICK-UP DEVICE

Michael Rettinger, assignor to Radio Corporation of America.
April 11, 1944, 7 Claims (Cl. 179–138).

Silk screens **32** and **34** fastened directly to the pole pieces of a ribbon microphone provide both acoustic



damping of the low frequency resonance point and protection against wind or explosive sounds.—FHS

2,348,356

5.9 MICROPHONE

Harry F. Olson, assignor to Radio Corporation of America.
May 9, 1944, 15 Claims (Cl. 179–138).

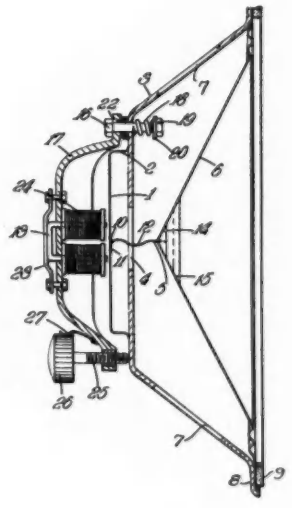
When a ribbon velocity microphone is operated at high magnetic flux densities, resistance reflected from the electrical circuit into the mechanical circuit alters the effective mechanical impedance of the ribbon. The reflected resistance changes the ribbon impedance from pure mass reactance to some non-linear function of frequency. If the microphone response is to remain flat under these conditions, the driving force obtained from the sound wave must also be a non-linear function of frequency. This patent describes the construction of velocity microphones embodying various types of baffles which introduce the required non-linearity. The operation of the baffles, as explained in the patent text, is not too clear. The statements, "It is obvious that at the lower frequencies, the impedance of the passages (*small holes in the baffles*) is relatively high," *and* "as the frequency . . . is increased, the impedance of the passages will be gradually decreased," seem rather surprising.—FHS

2,346,429

5.9 SOUND REPRODUCING AND RECEIVING DEVICE

Frank Harrison.
April 11, 1944, 2 Claims (Cl. 177–386).

This device is an iron armature type cone speaker. The



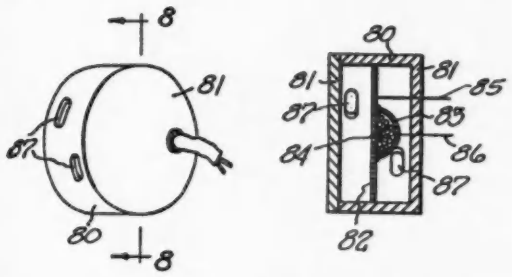
claims are quite restricted.—FHS

2,350,010

5.9 MICROPHONE

Francis Cheyney Beekley, assignor to the Glastonbury Bank and Trust Company.
May 30, 1944, 16 Claims (Cl. 179–1).

This is the patent on the well-known, "noise cancelling," lip microphone. The explanation given by the inventor says that sound waves from a distance enter the openings



87 and "act equally and oppositely upon the diaphragm 82, whereas sound waves applied in close spaced relation to one only of the openings 87 . . . will vibrate the diaphragm and actuate the microphone. In practice, such application of a spoken signal intended for transmission is accomplished by speaking directly into one only of openings 87 while the microphone is held close . . . to or even in contact with the lips of the user." "These openings 87 are spaced apart circumferentially a small distance, preferably not greatly in excess of $\frac{3}{8}$ inch."

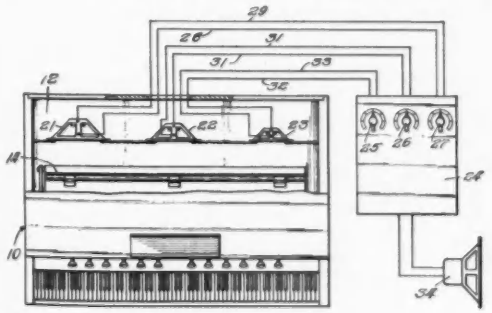
This overly simplified explanation implies, erroneously, that by careful balancing of the two sides of the microphone, background noise can be completely eliminated. —FHS

2,351,904

5.9 TRANSLATOR

Jesse Ora Ballinger and Edward B. Worthington, said Worthington assignor to said Ballinger.
June 20, 1944, 4 Claims (Cl. 179–115.5).

When bass notes of a reed organ are amplified to levels sufficient to fill a large auditorium, some of the higher partials may be irritating to the ear. To reduce the intensity of these overtones in the electrical reproduction the inventors have converted the conventional moving coil loudspeaker into a microphone having limited high frequency response. The diaphragm is heavy and made of blotting paper. The resilient spider has been replaced by an inelastic support of soft leather.—FHS

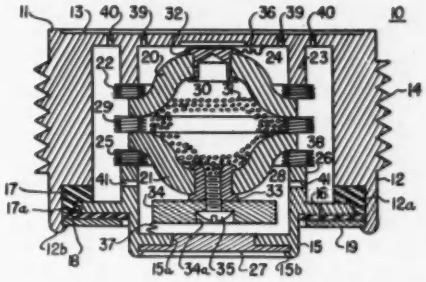


2,354,021

5.9 TRANSMITTER

Albert F. Horlacher, assignor to Automatic Electric Laboratories, Incorporated.
July 18, 1944, 11 Claims (Cl. 179–122).

Two movable electrodes 20 and 21 which enclose the carbon granules 31 in effect form two diaphragms. The sound pressure reaches diaphragm 20 through opening 39, and diaphragm 21 through openings 40 and 41. The dimensions of the various cavities and openings, the masses of



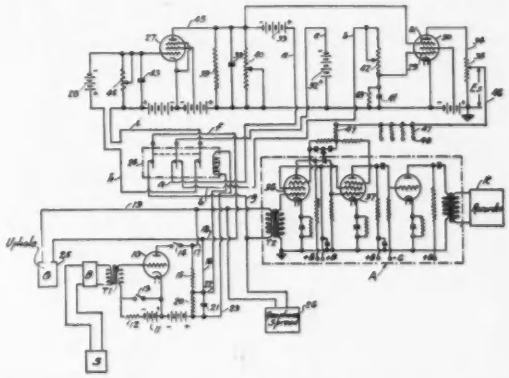
the various elements 33, 34, 35, 20, and 21 and the stiffness of the resilient supports 22, 25, and 29 are so proportioned that the microphone responds to a narrow band of frequencies. The important components of speech are supposed to lie within this pass band and the important components of noise are supposed to lie outside the band. —FHS

Re. 22,535

5.10 MASTER CONTROL FOR ELECTRIC SEISMOGRAPHS

Edwin J. Shimek, assignor, by mesne assignments, to Socony-Vacuum Oil Company, Incorporated.
August 22, 1944, 10 Claims (Cl. 177–352).

In a seismic prospecting system, an auxiliary geophone



is first connected to one of the channels and then discon-

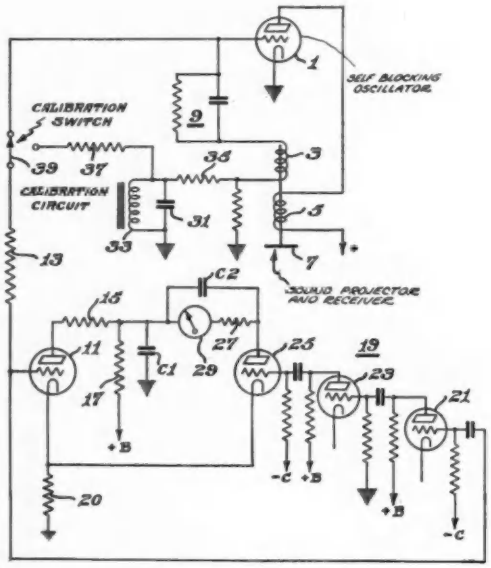
nected, the gain of all channels first reduced then increased in the proper sequence and at the chosen rates. Bias applied to suppressor grids affects the control of the gain.—GWD

2,346,093

5.10 SONIC DEPTH INDICATOR

William A. Tolson, assignor to Radio Corporation of America.
April 4, 1944, 5 Claims (Cl. 177–386).

"The operation of the device is as follows: The blocking oscillator **1** applies a current pulse to the field coils **3**, **5** of the sound projector which radiates a discrete sonic pulse or wave. The pulse is also applied positively to the control tube grid. This unblocks the control tube and permits current to flow through the resistor **15** and capacitor **C1** to ground to discharge the capacitor **C1**, whereupon the capacitor **C1** starts to charge up again through the resistor **17**. The controlling pulse also actuates the amplifier **19**, but the time constant of the amplifier is made long enough to prevent the amplifier output from interfering with the initial discharge of the capacitor **C1**. Furthermore, an additional bias for the amplifier output tube is derived from the resistor **20** in the control tube circuit.



"In the meantime, the radiated pulse travels toward the bottom or other reflecting object, and after reflection establishes a voltage in the sound receiver **3**, **5**, **7**. The received voltage pulse, being very much smaller than the transmitted voltage pulse, will not affect the control tube **11** but will be amplified sufficiently to unblock the amplifier output tube **25**. When the output tube is unblocked, current will flow through the meter and the timing capacitor will be discharged. The amplitude of the discharging current depends upon the time elapsing between the transmission or radiation of the wave and the reception of the reflected wave as indicated in the figure.

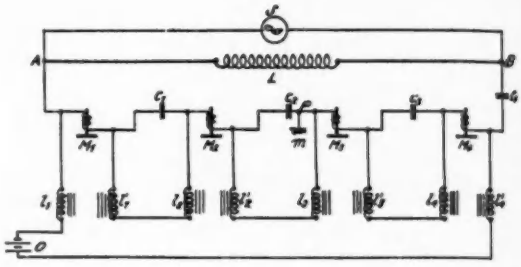
"The indicating meter **29** may be calibrated in terms of distance. The calibration will be logarithmic so that the readings will be increasingly crowded for the longer distances and increasingly spaced for the shorter distances. This type of scale is desirable in the depth indicator because shallow water will be indicated with greater accuracy than deep water, in which a high degree of accuracy is usually not required."—RWY

2,347,584

5.10 MARINE SOUND SYSTEM

Raymond Villem; vested in the Alien Property Custodian.
April 25, 1944, 3 Claims (Cl. 177–386).

"In a marine sound system of the character wherein sound waves are transmitted and received, the combination of a plurality of magnetostrictive microphones positioned to transmit sound waves of predetermined characteristics to the water and to be vibrated by sound waves from the



water, circuit means connecting said microphones in series and to a source of alternating current, a direct current circuit including a source of direct current and a plurality of blocking coils and connected to produce magnetic flux in each of said microphones with the blocking coils being effective to carry the direct current by preventing the passage of a substantial amount of alternating current, and a plurality of condenser units connected in said circuit respectively at said microphones and adapted to compensate for the phase displacements caused by the inductance of the microphones."—RWY

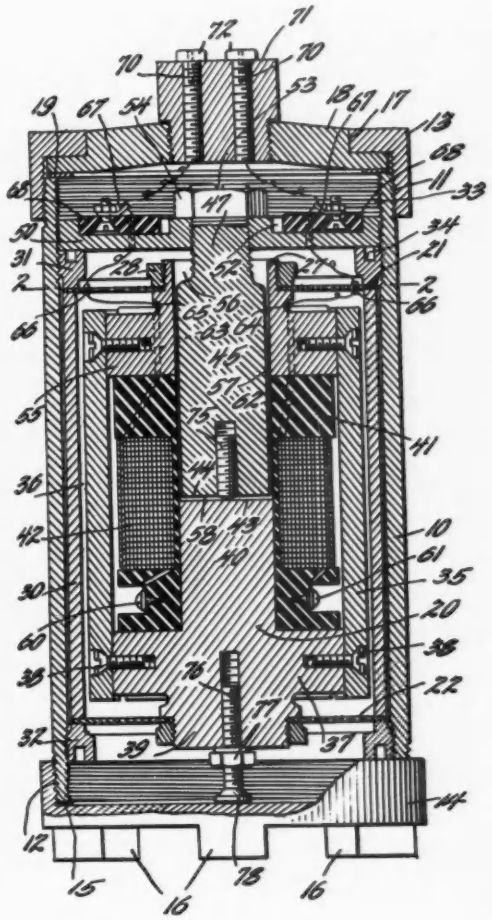
2,348,225

5.10 MAGNETIC SEISMOMETER

Olive S. Petty.
May 9, 1944, 7 Claims (Cl. 177–352).

The magnetic seismometer described in this patent generates an e.m.f. which is proportional to the vertical velocity component of the seismic wave. The steady mass in the system is composed entirely of such necessary parts

as the tubular permanent magnet **35**, pole pieces **40** and **55**, insulating spool **41**, and coil **42**. The soft iron armature



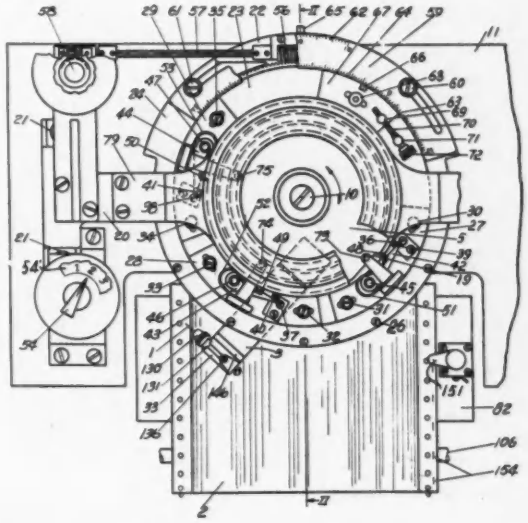
45 is fastened to the case. The steady mass is supported by the diaphragm springs **21** and **22**.—FHS

2,350,994

5.10 RECORDER

Albert E. Anderson, assignor to Submarine Signal Company.
June 13, 1944, 2 Claims (Cl. 117–386).

Among the features available in this echo sounding recorder are electrical selection (switch **54**) and remote indication of several depth scales, an adjustment (scale **65**) to correct for the depth of the echo sounding transducer, an adjustment (scale **63**) to make the record paper **2** direct reading in depth from some arbitrarily selected datum, a selection of paper speeds with remote indication



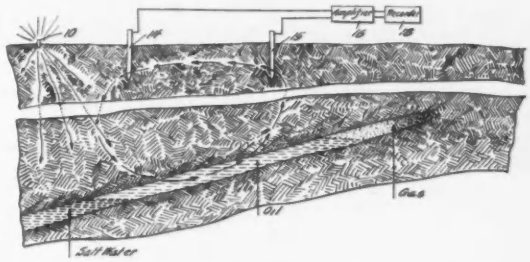
thereof, a fiducial mark entirely across the record, and time marks by secondary stylus **151**.—RWY

2,354,659

5.10 SEISMIC SURVEYING

Willis Oliver Bazhaw and Josephus O. Parr, Jr., assignors to Olive S. Petty.
August 1, 1944, 2 Claims (Cl. 181–0.5).

A means of seismic prospecting using potentials generated by flow of electrolyte through a porous medium as



the detecting means. This flow is caused by seismic waves from explosion.—GWD

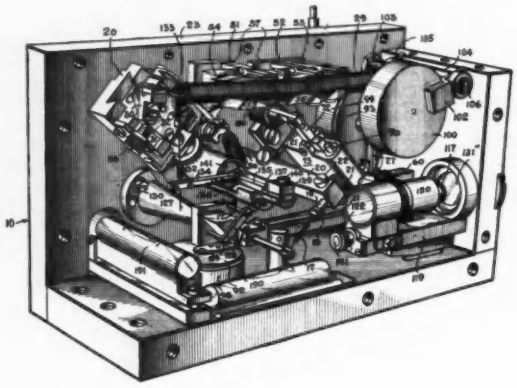
2,355,421

5.10 GRAVITY METER

Dayton H. Clewell and Henry A. Maeder, assignors to Socony-Vacuum Oil Company, Incorporated.
August 8, 1944, 17 Claims (Cl. 265–1.4).

A gravity meter is described in considerable detail. An extensive analysis is given to show that this design is par-

ticularly such that the instrument is relatively insensitive to tilt but is very sensitive to changes in gravity.—GWD

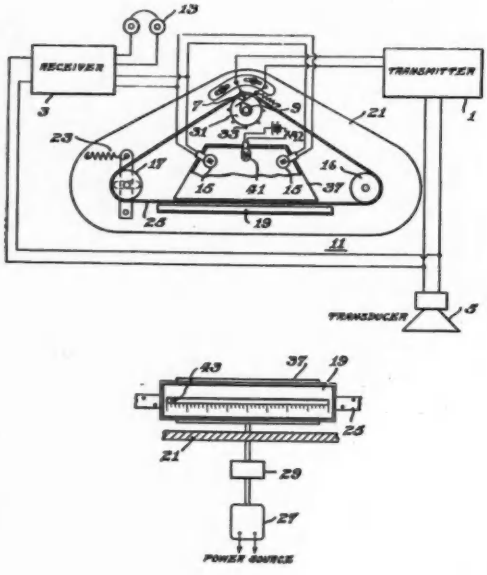


2,355,502

5.10 SIGNAL INDICATOR AND RECORDER

Loy E. Barton, assignor to Radio Corporation of America. August 8, 1944, 6 Claims (Cl. 177–386).

"A transmitter **1** and a receiver **3** are connected to a transducer or sound projector **5**. The transmitter is preferably of the pulse type and is keyed by a switch **7** operated by a cam **9** in the indicator or timing device **11**. The receiver **3** is connected to apply the received signals to



telephones or other aural indicator **13** and to gaseous discharge tubes **15** or other quick operating light valves in the timing device.

. . . . .

"The operation of the device is as follows: The motor **27** drives the pulley **33** at constant speed so that the belt moves at a given speed along the longitudinal axis of the scale **19**. The index aperture **43** is arranged to pass the zero of the scale at the instant the cam switch **7** pulses the transmitter **1**. The outgoing pulse is radiated from the diaphragm of the transducer **5**. The pulse travels at a known velocity through the medium and returns, after reflection, to set up a signal current in the transducer. . . . The detected signal produces an aural signal in the telephones **13** and flashes the gaseous discharge lamps **15**. During the propagation of the signal the belt aperture **43** will move along the scale **19**, which may be calibrated to indicate distance in any desired units, and its image will appear at the instant the signal is received.

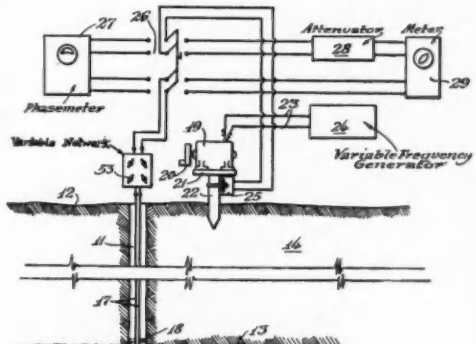
"If the incandescent light has been turned on by closing a switch **42** in circuit therewith, the aperture will be illuminated continuously and its image will move along the scale as a timing mark. The operator may then either observe the changing of the color of the image due to the gaseous discharge or, if the signal is not sufficient to flash the lamp **15**, the operator may associate the aural signal with the indicator position to determine the distance."—RWY

2,355,826

5.10 SEISMIC PROSPECTING

Joseph A. Sharpe, assignor to Stanolind Oil and Gas Company.
August 15, 1944, 5 Claims (Cl. 177–352).

This patent covers a method for determining the wave transmission characteristics of the weathered layer. The



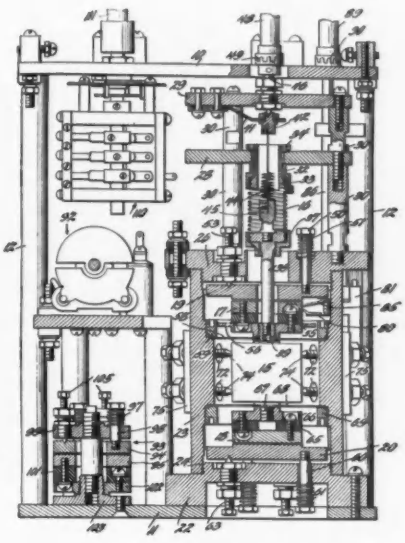
variable network **53** is adjusted to have loss and phase shift characteristics conjugate to that of the path between source **21** and geophone **16**. Networks of this characteristic are inserted in each of the channels of a conventional seismograph system to produce a correction for the weathered layer—GWD

2,357,356

5.10 MEASURING INSTRUMENT AND METHOD

Olive S. Petty.
September 5, 1944, 10 Claims (Cl. 265–1.4).

This invention relates to certain improvements to a gravimeter used in geophysical exploration. Means are

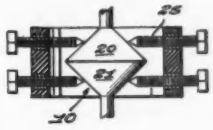


described for affecting temperature compensation of the suspension and the indicating system.—GWD

2,357,822

5.10 MEASURING INSTRUMENT

Arthur F. Hasbrook, assignor to Olive S. Petty.
September 12, 1944, 12 Claims (Cl. 265–1.4).



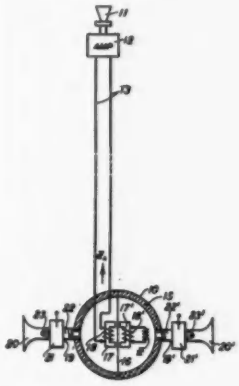
In this invention a method is described for constraining the motion of the mass in a gravimeter without introducing errors into the system. Pointed contacts **25** are located in close proximity to the mass **10** having conical surfaces **20** and **21**. Motion greater than the clearance is prevented. Absence of contact is determined electrically to insure absence of constraint during measurements.—GWD

2,348,629

5.13 PUBLIC ADDRESS SYSTEM

Kenneth S. Johnson, assignor to Bell Telephone Laboratories, Incorporated.
May 9, 1944, 6 Claims (Cl. 179–1).

It is claimed that by adjusting the positions of the two microphone mouthpieces **20** and **20′** and the acoustic impedances **21** and **21′**, the sound from the loudspeaker **11** can be made to produce no pressure difference between the two sides of the microphone diaphragm **16**. Thus acoustic feedback would be minimized. The sound intended to be amplified and radiated from the loudspeaker can be introduced into only one of the microphone mouthpieces—



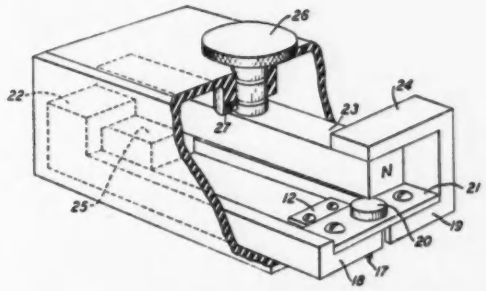
supposedly without seriously upsetting the antifeedback balance.—FHS

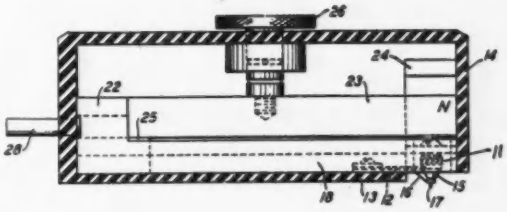
2,347,347

5.16 VIBRATION TRANSLATING DEVICE

George R. Yenzer, assignor to Bell Telephone Laboratories, Incorporated.
April 25, 1944, 5 Claims (Cl. 179–100.41).

Both vertical and lateral type recordings may be reproduced with this pick-up. The change in operation is effected merely by moving the permanent magnet **23** from the lower to the upper position. Stylus **17** with its supporting member **12** may vibrate either axially or torsionally. Coil **11** is enclosed in an annular air gap formed by pole pieces, **18**, **19**, and **20**, **21** being non-magnetic. Pole piece **19**





is U-shaped in front and L-shaped to the rear, terminating in arm **25**. The radial field required for reproduction of vertical records is obtained by having the magnet **23** in contact with pole piece **20** and arms **22** and **25**; the trans-

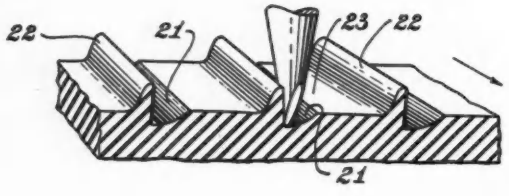
verse field required for reproducing lateral records is obtained by having the magnet in contact with pole piece extension **24** and arm **22** only.—LWS

2,347,623

5.16 SOUND RECORD AND METHOD OF MAKING

Robert Wagner, assignor to The Dow Chemical Company.
April 25, 1944, 8 Claims (Cl. 274–46).

"It is among the objects of the invention to provide a method of recording sound, particularly by lateral modulations, whereby the shavings, cuttings, and other debris incident to known practice are eliminated . . . (and to produce) . . . a sound track . . . capable of providing substantial lateral support and contact surface for reproducing needles." This is accomplished by employing a



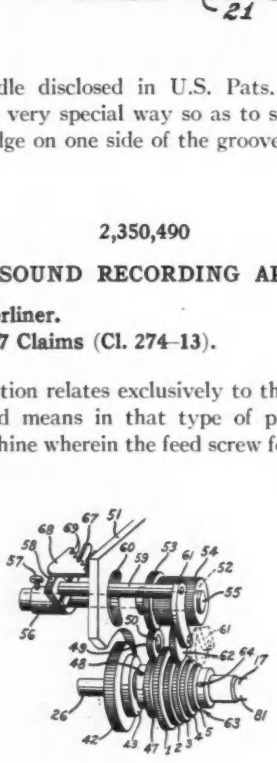
recording needle disclosed in U.S. Pats. 1,918,271 and 2,018,496 in a very special way so as to slit the material and form a ridge on one side of the groove.—LWS

2,350,490

5.16 SOUND RECORDING APPARATUS

Herbert S. Berliner.
June 6, 1944, 7 Claims (Cl. 274–13).

"This invention relates exclusively to the incorporation of change feed means in that type of phonograph disc recording machine wherein the feed screw for the recording



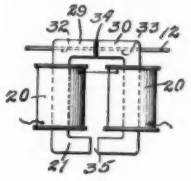
head is driven in a positive manner from the top of the recording machine turntable." By moving lever **67** in index plate **69**, idler gear **62** is engaged with various step gears **1** to **5**.—LWS

2,351,003

5.16 RECORDING AND REPRODUCING OF VIBRATIONS

Marvin Camras and William Korzon, assignors to Armour Research Foundation.
June 13, 1944, 15 Claims (Cl. 179–100.2).

With the usual arrangement for recording on magnetic wire, the wire is magnetized unsymmetrically circumferentially, so that any twisting of the wire under the reproducing pole relative to the recording pole results in serious distortion. To overcome this defect and also to



reduce distortion due to fringing flux, the wire is passed through a hole **32** extending through the pole pieces **29** and **30**. The gap **34** is filled with a non-magnetic material which also serves to exclude dirt. Since the interior of the pole pieces is magnetically neutral demagnetization caused by fringing flux is eliminated.—LWS

2,351,004

5.16 METHOD AND MEANS OF MAGNETIC RECORDING

Marvin Camras, assignor to Armour Research Foundation.
June 13, 1944, 11 Claims (Cl. 179–100.2).

According to this invention it is possible to use a magnetic wire medium for recording purposes in the demagnetized state and to obtain a linear characteristic curve by superimposing the signal to be recorded upon a high frequency carrier. Figure 1 shows the wave presented to

FIG. 1

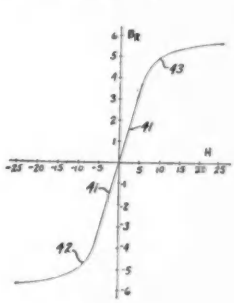


FIG. 2

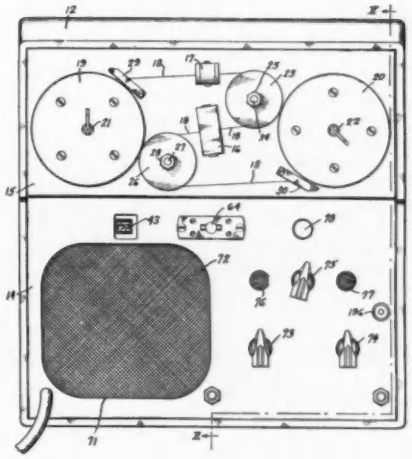
the recording head and Fig. 2 the consequent linear residual magnetization curve resulting from the process. It is stated that this method increases the dynamic range, reduces distortion, and makes possible simultaneous erasing of previous recordings.—LWS

2,351,005

5.16 MAGNETIC RECORDER

Marvin Camras, assignor to Armour Research Foundation. June 13, 1944, 4 Claims (Cl. 179–100.2).

This patent relates to a magnetic recording and reproducing machine embodying features of patents 2,351,003 and 2,351,004 together with means for switching and



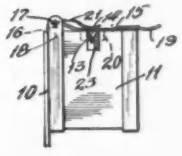
indicating, driving and spooling the wire. The same head **16** is used for both recording and reproducing, the high frequency oscillator being disconnected in the latter case. —LWS

2,351,006

5.16 MAGNETIC RECORDING HEAD

Marvin Camras, assignor to Armour Research Foundation. June 13, 1944, 11 Claims (Cl. 179–100.2).

In conjunction with a magnetic wire recorder such as disclosed in preceding patents, it is often desirable to be able to remove the wire or splice it by means of a knot



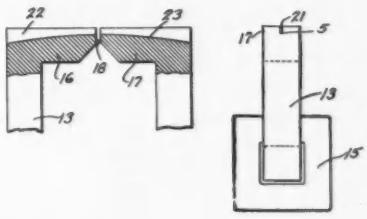
which will pass through the recorder. This is accomplished by providing said recording head with a narrow slot **13** for passage of the normal wire and a wider slot **14** for the knot or splice, there being a gradually tapered entrance to the head so that the knot will automatically find a portion of the slot large enough to pass it.—LWS

2,351,007

5.16 MAGNETIC RECORDING HEAD

Marvin Camras, assignor to Armour Research Foundation. June 13, 1944, 12 Claims (Cl. 179–100.2).

Wire only a few thousandths of an inch in diameter may be conveniently threaded into this magnetic recorder head. The claims cover a straight slot as shown in the



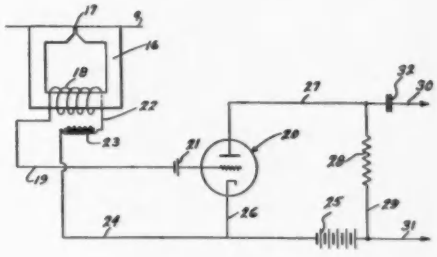
figures as well as bayonet or L-shaped slots. Also claimed are poles of unequal length intended to minimize the effect of stray fields.—LWS

2,351,008

5.16 METHOD OF AND MEANS FOR NEUTRALIZING INDUCTIVE DISTURBANCES IN MAGNETIC REPRODUCERS

Marvin Camras, assignor to Armour Research Foundation. June 13, 1944, 5 Claims (Cl. 179–100.2).

This patent teaches that stray pick-up induced in the reproducing head of a magnetic reproducer may be



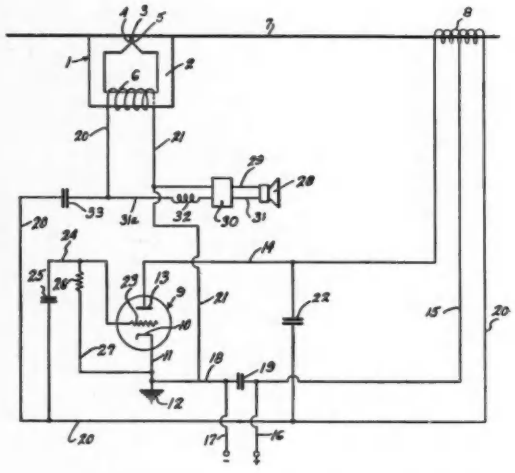
eliminated by connecting in series with same an auxiliary coil so oriented as to neutralize the noise induced in said reproducer head.—LWS

2,351,009

5.16 COMBINATION OSCILLATOR COIL AND ERASING HEAD FOR MAGNETIC RECORDERS

Marvin Camras, assignor to Armour Research Foundation. June 13, 1944, 5 Claims (Cl. 179–100.2).

Disclosed in this patent is a way of combining the erasing coil in a magnetic recorder with the frequency determining



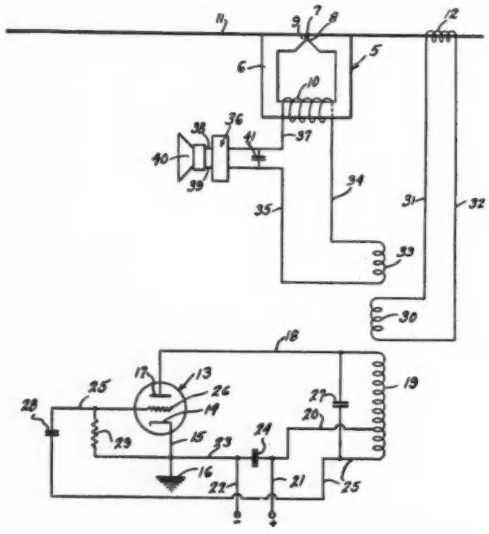
coil in the oscillator required by the method of patent 2,351,004.—LWS

2,351,010

5.16 METHOD OF AND MEANS FOR CONTROLLING HIGH FREQUENCY VOLTAGE IN MAGNETIC RECORDER HEADS

Marvin Camras, assignor to Armour Research Foundation. June 13, 1944, 11 Claims (Cl. 179–100.2).

This patent relates to a means of controlling the high frequency current supplied to a magnetic recording head. Both coil **30**, which feeds erasing coil **12**, and coil **33**, which feeds coil **10**, are inductively coupled to oscillator coil **19**,



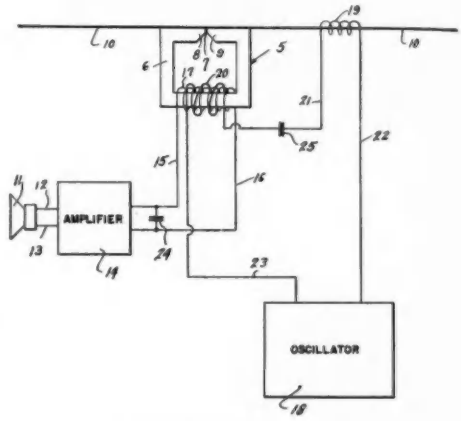
the polarity of **30** being so chosen that the current induced in it will produce a field opposing that of coil **19**. Thus by adjusting the impedance of the circuit of coil **30**, as by adjusting the number of turns of coil **12**, the current in coil **10** may be controlled.—LWS

2,351,011

5.16 METHOD OF AND MEANS FOR ENERGIZING MAGNETIC RECORDER HEADS

Marvin Camras, assignor to Armour Research Foundation. June 13, 1944, 8 Claims (Cl. 179–100.2).

This patent is a continuation of patent 2,351,110 and covers the inductive coupling of the high frequency and



audio currents in the recording head. The strength of the former is controlled by adjusting the impedance of the oscillator circuit.—LWS

2,351,600

5.16 LAMINATED SOUND RECORD

William R. Collings, assignor to The Dow Chemical Company.
June 20, 1944, 7 Claims (Cl. 154–43).

This patent discloses a method of producing a laminated phonograph record from ethyl cellulose of specified composition, wherein **13** is a hard heat-resistant layer, **12** is a thermally cementitious layer, and **11** is a core which may

be paper or metal. In the manufacturing process the layers **12** and **13** blend together so that there is no line of demarcation between them. The filler core may have printing thereon without affecting the bond.—LWS

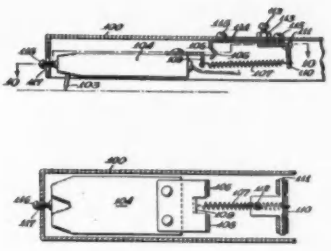
2,351,948

5.16 SOUND RECORDING AND REPRODUCING APPARATUS

Paul Stead Gay, assignor to The Wilcox-Gay Corporation. June 20, 1944, 11 Claims (Cl. 274–13).

A particular feature of this recording machine, which has been designed to be operated by unskilled, non-technical people and for manufacture by mass production

methods, is the mounting of the recorder and reproducer. For example, the reproducer is pivotably secured by knife edge projections **105** which seats in V-grooves **106** owing to tension of spring **107**. Said tension is adjustable by means



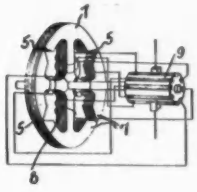
of screw **112** acting on bracket **110** which is pivoted at **111**. It is claimed that such an arrangement provides the proper stylus pressure regardless of record and turntable wobble. —LWS

2,352,023

5.16 SOUND REPRODUCER

Eduard Schuller.
June 20, 1944, 5 Claims (Cl. 179–100.2).

According to this invention magnetically recorded sound may be reproduced at a speed different from that at which it was recorded without changing the original pitch of the



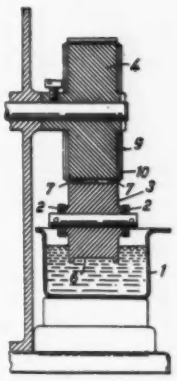
sound. This is accomplished by rotating the reproducing head over which the magnetic tape is drawn so as to maintain the original relative velocity between the tape and the recording point. It is stated that the spacing between the reproducer gaps **5** should be less than one-tenth the distance traveled by the magnetic carrier in one second during recording.—LWS

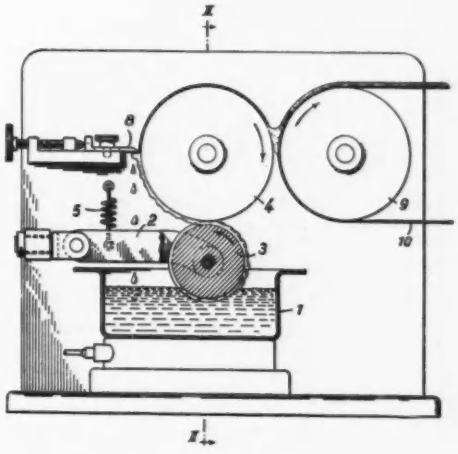
2,352,052

5.16 METHOD OF PRODUCING SOUND BAND MATRICES

Hugo Westerkamp; vested in the Alien Property Custodian.
June 20, 1944, 6 Claims (Cl. 18–57).

This patent relates to a method of producing a matrix of a mechanically recorded sound band **10** by coating the surface with a gelatinous substance. In the process the





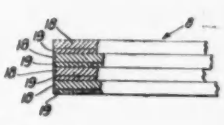
drum **4** to which said substance is transferred does not come into direct contact with the sound band but a damming action which takes place at the interface between the roller and the sound band causes the transfer.—LWS

2,352,285

5.16 METHOD OF MAKING PHONOGRAPH RECORD STAMPERS

Barton A. Proctor, assignor to B. A. Proctor Company, Incorporated.
June 27, 1944, 4 Claims (Cl. 154–2).

This patent covers a method of producing phonograph record stampers comprised of a laminated structure of



paper disks **18** impregnated with a thermosetting plastic **19** mixed with metallic powder to inhibit flow of the plastic and to increase the heat transfer.—LWS

2,356,145

5.16 MAGNETIC SOUND RECORDING AND REPRODUCING

Semi Joseph Begun, assignor to Magnetone Incorporated.
August 22, 1944, 28 Claims (Cl. 179–100.2).

A principal feature of this invention is a synchronized switching means whereby sound may be recorded on or reproduced from any desired portion of an endless or coiled magnetic carrier. This feature is said to be desirable as a teaching aid since it is possible for the student of language to compare his recording easily with a model record.—LWS

2,356,262

5.17 ACOUSTIC DEVICE

Edward E. Mott, assignor to Bell Telephone Laboratories, Incorporated.
August 22, 1944, 9 Claims (Cl. 179–180).

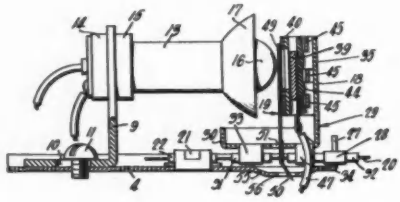
The specifications in this patent are so complete that it is, in effect, a textbook on the design of acoustically damped telephone receivers. Although the main body of the patent considers the magnetic structure as well as the acoustical circuit of such receivers, the claims seem to be concerned largely with the acoustical circuit.—FHS

2,348,660

5.19 PHONOGRAPH

Ingo L. Stephan, assignor to Philco Radio and Television Corporation.
May 9, 1944, 8 Claims (Cl. 179–100.41).

A principal feature of this electro-optical phonograph



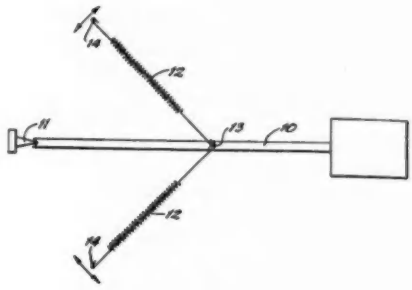
reproducer is the resiliently mounted cantilever support **20** for the stylus **50** and light gate **49**.—LWS

2,343,063

5.21 METHOD OF AND APPARATUS FOR MEASURING MECHANICAL VIBRATIONS

Earle L. Kent, assignor to C. G. Conn, Limited.
February 29, 1944, 15 Claims (Cl. 73–51).

"The underlying principle of the invention is a mechanical system which is resonant or can be made to be resonant at the frequency of vibration it is desired to measure. . . . As shown in the figure, the system may comprise an elongated vibratory member **10** pivoted at **11** and adopted to oscillate about **11** in response to a vibration impressed at the pivot point." Adjustment of the resonant frequency is accomplished by moving the supporting pins **14** on arcs such as to maintain the initial tension on the springs **12**,



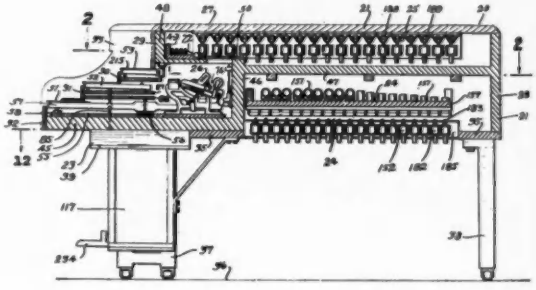
yet changing the direction of their restoring force. The amplitude of vibration is read from a suitably calibrated scale, damping having been provided of such a character that at resonance the amplitude is independent of frequency.—RWY

2,351,497

6.1 MUSICAL INSTRUMENT

Basil N. Fomeen.
June 13, 1944, 2 Claims (Cl. 84–170).

This musical instrument, "The Basifon Grand," puts within reach of the player a piano **22** (keyboard **51**), a reed organ **23** (keyboard **52**), a vibraphone **24** (key board **53**), tubular chimes **25**, cymbalum effect mechanism **26**. The last consists of a metal strip carrying an enlargement



74 which can be lifted into contact with the strings, and between them and the hammers, to produce a buzzing sound. Since the keyboard for the chimes is short it is at the same level as that for the vibraphone.—RWY

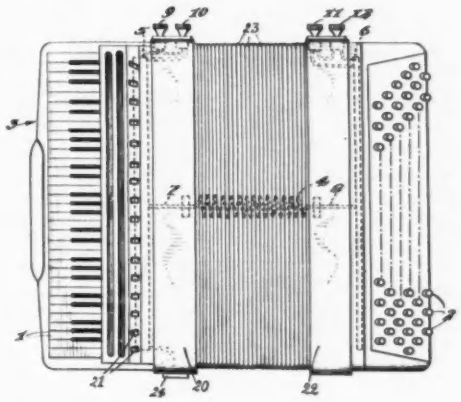
2,356,368

6.1 MUSICAL INSTRUMENT

Dominick Zaccagnino.
August 22, 1944, 1 Claim (Cl. 84–376).

Long microphones **5** and **6** with independent tone and volume controls **9**, **10**, **11**, and **12** are arranged in this accordion in positions intended to allow uniform pick-up

from all reeds. The two microphones are interconnected by wire **4** so that only a single connection to the external



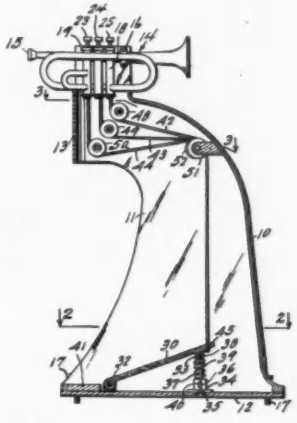
amplifier is required.—RWY

2,358,605

6.3 FOOT OPERATABLE ATTACHMENT FOR VALVED WIND INSTRUMENTS

Wiley H. Strane.
September 19, 1944, 8 Claims (Cl. 84–388).

"This invention, a foot-operated attachment for wind instruments, is designed specifically for valve-controlled instruments, such as cornets, trumpets, key trombones, altos, basses, and french horns, and as illustrated is applied



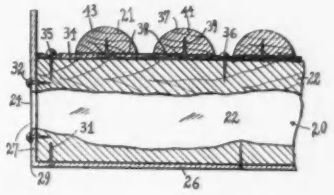
to a trumpet. This invention makes possible the playing of such instruments by those who have lost the use of their hands for operating the valves for playing the instruments, with operation being conducted by the feet of the player." —RWY

2,347,287

6.4 XYLOPHONE

Elmer Sas.
April 25, 1944, 6 Claims (Cl. 84–403).

In this xylophone the bars of semicircular section are attached to a cord **34** running along under the nodes of the vibrating bars. The supporting cord is in turn fastened to



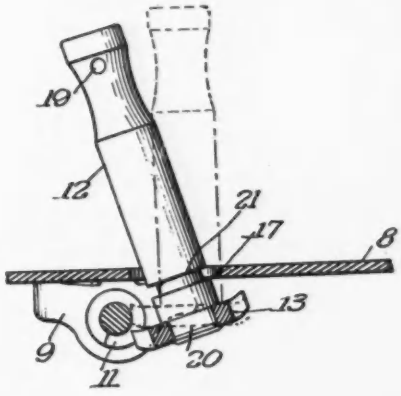
the side rail **22** as by nails **36**. This allows the bars to be relatively free of the rigid frame.—RWY

2,356,766

6.7 TUNING HEAD FOR STRINGED INSTRUMENTS

John E. Kluson.
Augist 29, 1944, 1 Claim (Cl. 84–306).

This tuning pin for a stringed musical instrument, such as a guitar, is so designed that it may be installed with the



gear **13** already attached. The hole **17** in plate **8** is of such shape as to receive the grooved portion **21** of pin **12** and thus to limit the vertical motion of the pin. Under the tension of the string the pin assumes the dotted position and gear **13** is meshed properly with the driving worm **11**. —RWY

2,357,191

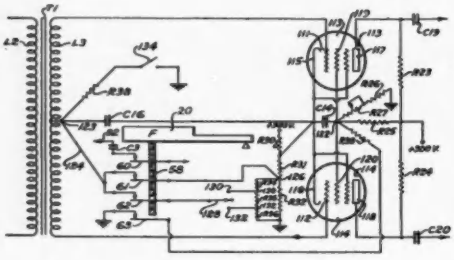
6.9 ELECTRICAL MUSICAL INSTRUMENT

John M. Hanert, assignor to Hammond Instrument Company.
August 29, 1944, 30 Claims (Cl. 84–1.26).

This solo type electrical musical instrument may be played by means of a short pedal clavier and may thus be used conveniently in conjunction with other instruments such as piano or accordion. If no keys are depressed the signal impressed upon tubes **113, 114** is not amplified because screen grids **119, 120** are negative relative to the

cathodes **115, 116.** With switch **134** closed and **128** connected to **132**, as key **F20** descends switch **60** opens thereby connecting capacitance **C3** into the circuit which establishes the frequency of the sound to be produced. At the same time, **61** opens thus permitting the center tap **123** to drop toward the potential of **132**. The rate at which this change occurs depends upon **C16** and **R36**.

"After a time interval determined by the velocity of key depression, the switches **62** and **63** are opened. Upon opening of the switch **62** the change of potential of the center



tap **123** toward the potential of the terminal **132** ceases. Opening the switch **63** results in the screen potential of the terminal **122** rising to its normal operating value of approximately 175 volts, thus permitting current to flow to the plates **117, 118** under the control of the signal on the control grids **111** and **112**.

"It will thus be apparent that the amplitude of the plate current is variable, depending on the control grid bias, the latter being a function of the time interval between the opening of the contacts **61** and **62**, which in turn is a measure of the velocity with which the pedal key **F** is depressed. After the key has been fully depressed, further reduction in the control grid bias occurs through the discharge of condenser **C16** through resistance **R38** which is connected to ground through closed switch **134**. As this condenser slowly discharges to ground, the effect of a slowly decaying percussion tone is produced."

The system is less responsive to touch when switch **128** goes to **130** and not at all touch responsive if **128** is left in the intermediate position. If **134** is open no decay occurs while the key is depressed.—RWY